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Probability in the Talmud

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SUMMARY

Typical examples of probability arguments are adduced from the Talmud to show that the ancient Rabbis formulated rules for adding, multiplying and comparing probabilities as well as criteria for sampling and estimating populations.

1. INTRODUCTION

Hasofer (1966) has reviewed some occurrences in the Talmud of the use of random mechanisms to secure a 'fair' outcome. In the present paper, probabilistic notions in the Talmud are investigated particularly as they occur in situations other than those involving a deliberate use of chance. Questions involving probability considerations occur frequently in the ancient rabbinic literature, and we shall show that the Rabbis computed probabilities in accordance with certain preconceived logical principles.

There are, in fact, two Talmuds. They consist of (*a*) a common basic text, the *Mishnah*, substantially completed in the second century, and (*b*) the *Gemara*, the discussions of the Rabbis on the text of the *Mishnah*. The *Jerusalem Talmud* was brought to essentially its present form by the end of the fourth century, the *Babylonian Talmud* about a century later. References to Talmudic texts are given in the standard form, i.e. tractate, folio number and side for the Babylonian Talmud, and, for the Jerusalem Talmud, tractate, chapter and section, just as for the Mishnah.

For a general introduction to the Talmuds, see Strack (1945).

2. 'FOLLOW THE MAJORITY'

A typical question involves objects whose identity is not known and reference is made to the likelihood that they derive from a specific type of source in order to determine their legal status, i.e. whether they be permitted or forbidden, ritually clean or unclean, etc. Thus, only meat which has been slaughtered in the prescribed manner is *kasher*, permitted for food. If it is known that most of the meat available in a town is *kasher*, there being, say, nine shops selling *kasher* meat and only one that sells non-*kasher* meat, then it can be assumed when an unidentified piece of meat is found in the street that it came from the majority and is therefore permitted.

However, the Talmud distinguishes between an enumerated majority and one that is just taken for granted *a priori*.

(Hullin 11*a*): Whence is derived the rule which the Rabbis stated: 'Follow the majority'?...As for a majority that is enumerated as in the case of the Nine Shops or the Sanhedrin we do not ask the question. Our question relates to cases where the majority is not explicit, as in the case of the Boy and Girl [who we assume will grow up to be fertile and are therefore not to be considered eunuchs with

respect to levirate marriage, although this cannot now be ascertained. The reason is that we follow the majority, and the majority of people are not sterile. Yet, this majority is not an explicit one since we cannot count all the people].

Not all the sages agreed to follow a majority when that is not actually enumerated. Rabbi Meir and others 'are concerned for the minority' (Gittin 2b), except where the uncounted majority is overwhelming. Such is the presumption that most of 'the scribes of the courts know the law' (Gittin 2b), for an ignorant scribe must be so rare that a legal document issued by the court may be assumed to have been properly executed.

Even for explicit majorities, it is important to determine whether the majority is relevant to the case in question.

(Kethuboth 15a): All that is stationary (fixed) is considered as half and half...If nine shops sell ritually slaughtered meat and one sells meat that is not ritually slaughtered and he bought in one of them and does not know which one; it is prohibited because of the doubt; but if meat was found [in the street], one goes after the majority.

The reasoning seems to be that when the question arises at the source, the chances are not really nine to one. For the other nine shops do not enter into the picture at all, since the piece of meat in question certainly does not come from any of them. Therefore there are only two possibilities and the chances of its being *kasher* or not must be considered even. However, if meat is found in the town at large, the chances that it comes from any one of the ten shops are equal and therefore the probability that it is *kasher* is 9/10. This is expressed in the rule: 'that which is detached, is detached from the majority'.

3. ADDITION AND MULTIPLICATION OF PROBABILITIES

If a husband dies, leaving his wife pregnant, the probability that she will bear a live male child is a factor in the application of certain laws of inheritance. This probability must be less than one-half. For,

(Yevamoth 119a): A minority [of pregnant women] miscarry and of all the live births half are male and half female. Add the minority of those who miscarry to the half who bear females and the males are in a minority.

A 'minority of a minority' is defined in a discussion concerning cheese made with rennet derived from the stomachs of cattle.

(Avodah Zarah 34b): Since there would be a majority of calves that are not slaughtered for idolatry and then there are the other [adult] cattle [of whom none are slaughtered for idolatry and they are a majority as against all calves] it would be a minority of a minority and even Rabbi Meir is not concerned for a minority of a minority.

An interesting example is the following, where equal probabilities are multiplied to obtain a minority. This is called a 'doubt of a doubt' or 'double doubt', and is distinguished from a minority of a minority.

An adulteress is forbidden to live with her husband, so that her husband may divorce her without penalty. But this is not so if he claims that his bride committed adultery before the consummation of the marriage, even though it be established that she is not a virgin.

(Kethuboth 9a): It is a double doubt. It is a doubt: whether under him [i.e. during the period—usually one year—between formal betrothal and consummation] or not under him [i.e. prior to the betrothal]. And if you say that it was under him there is the doubt whether it was by violence or by her free will.

It is assumed that the chances are at most even that the incident occurred during the period of betrothal. But even if such were the case, since a woman violated by another man

does not become forbidden to her husband, it can be argued that there is at least an even chance that she did not submit of her own free will. Thus the probability that the husband's claim is correct is only $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

The commentators distinguish between a 'reversible' double doubt such as the above, where the probabilities are independent, i.e. there are four possible cases, and a 'non-reversible' one where the second doubt is conditional on the first so that there are only three distinct possibilities.

4. COMPARING PROBABILITIES

Ordinarily, when a forbidden object falls into a majority of similar objects that are permitted, the prohibition is annulled, so that if one draws any object it is permitted (Zevahim 72a).

However, the prohibition of objects used in idolatrous worship is so grave that 'even one in ten thousand' makes them all prohibited (Zevahim 74a). From that discussion we quote:

If an idol's ring was mixed up with [others making a total of] one hundred rings and forty of them were separated to one place and sixty to another...the forty detached to one place do not render others forbidden: the sixty in one place do render others forbidden.

Although the odds are 3:2 that the idol's ring is in the sixty, this is not sufficient reason to permit drawing from the forty, since on an individual draw from either group the probability is the same, 1/100, that the forbidden ring is chosen. Thus the remark in the same source:

Why is one from the forty different? [Presumably] because we say, The forbidden one is among the majority. Then in the case of one from sixty too we must say, The forbidden one is in the majority [i.e. the remaining fifty-nine]!

However, when at least one additional ring is added to each group the probabilities are no longer equal for an individual draw from each set. This is the meaning of the rule 'the forty... do not render others forbidden'.

While no computations are given in the text, it is remarkable that the division of 40:60 is chosen rather than 49:51 which would make a stronger point (cf. Pesahim 79b ff.). However, a calculation of the different probabilities is revealing, and we can attempt to reconstruct what might be a plausible line of Talmudic reasoning.

Suppose to the set of X rings ($X < 50$), separated from the original mixture, is added at least one more ring (for the worst case) to make the set A. Similarly to those remaining from the original mixture add at least one to form the set B.

The probability p_A of drawing the idol's ring from the set A is

$$p_A = \frac{X}{100} \frac{1}{X+1}, \quad \text{while} \quad p_B = \frac{100-X}{100} \frac{1}{101-X}.$$

Clearly $p_B > p_A$.

Elsewhere, the Rabbis specify for an effective majority that it exceed the minority by a certain minimum (e.g. Berakhoth 48a, Sanhedrin 2a, Hullin 28a, etc.). Thus, $(p_B - p_A)/p_A$ can be considered as an indicator of the greater likelihood that a single draw from B, rather than from A, gives the forbidden ring, and this must be of significant magnitude. We seek a lower bound for this ratio.

Now for large X, both p_A and p_B are close to 1/100; and, since 1 in 10,000 renders a mixture unfit, a probability of 1×10^{-4} relative to 1×10^{-2} is still decisive, i.e. if $(p_B - p_A)/p_A$ is near $1 \times 10^{-4}/1 \times 10^{-2}$, the set A can be permitted. In fact, the minimum significant probability

must be somewhat less than 1×10^{-4}. Therefore the upper bound for X should not be the least value such that

$$L(X) = \frac{p_B - p_A}{p_A} \frac{1}{100} < \frac{1}{10,000}.$$

It suffices to choose the next larger X.

We get the following values:

X	38	39	40
$L(X)$	1/9975	11/120,900 $\simeq$ 1/10,991	1/12,200

Therefore $X = 40$ is chosen as the maximum admissible value for which the set A can be permitted.

5. SAMPLING

The problem of drawing a typical example from a population is raised in several contexts. In general, there are two views; one maintains that two instances indicate a pattern, while according to another view, three are required, but there are some variations.

Tefillin are ritual objects which must be prepared according to exacting stipulations. One is worn on the arm and another on the head.

(Eiruvin 97a): If one acquires [bundles] of Tefillin from one who is not certified, he checks two for the arm and one for the head or two for the head and one for the arm...similarly, for the second and third bundles...Since each bundle was made by a [different] person...the third one is mentioned to show that there is no presumption with respect to the bundles...and even the fourth and fifth bundles [require sampling].

Thus it appears that a sampling is required to establish the reliability of each individual source. However, in the Jerusalem Talmud the question is left open;

(Eiruvin X—1): If one found two or three bundles, he checks a pair from the first bundle and similarly from the second and third. Isaac ben Elazar asked: Is there one pattern for all of them or must it be for each one separately? If you say one pattern for all of them, he checks the first pair from the first bundle, etc.; if you say each one has its own pattern, he checks three pairs in each bundle.

In dealing with major physical abnormalities, the Talmud rules that two recurrences of a phenomenon are to be considered a significant indication of variation. Rabbi Judah the Prince (2nd cent.) ruled

(Yevamoth 64b): [A mother] had one child circumcised and he died; a second one and he died; one must not circumcise the third.

The same rule applies if the children were not born to the same woman but the mothers are sisters for 'sisters establish a pattern' (*ibid.*). However, if there is only one unusual occurrence, it may be mere coincidence (cf. Kethuboth 87b).

To this day, in the absence of medical proof that the deaths are unrelated, Rabbi Judah's rule applies.

The size of the sample relative to the population was specified in the following instance.

(Taanith 21a): A town bringing forth five hundred foot-soldiers like Kfar Amiqo, and three died there in three consecutive days; it is a plague...A town bringing forth one thousand five hundred foot-soldiers like Kfar Akko, and nine died there in three consecutive days; it is a plague.

There are, however, no indications what was the proportion of foot-soldiers to the total population.

6. INCONCLUSIVE PROBABILITIES

Legal principles are applied when the probabilities are inconclusive.

An offering called *Terumah* is to be set aside from the grain crop. This is sacred and may be eaten only by priests, so that a mixture of *Terumah* and ordinary grain becomes forbidden to all except priests. However,

(Mishnah Terumoth VII–5): Two bins, one containing *Terumah* and one ordinary grain, and a bushel of *Terumah* fell into one of them but it is not known into which one; I say that it fell into the *Terumah*.

Thus the ordinary grain remains permitted. None the less there is a difference of opinion whether the same rule applies in the following circumstances:

(Yevamoth 82a): Two bins, one containing *Terumah* and one ordinary grain, and before them are two bushels one of ordinary grain and one of Terumah and the latter fell into the former, one into each.

Since it is known that each bushel fell into a different bin, the events are not independent and the probability is just 1/2 that the Terumah fell into the ordinary grain. The commentators explain that the rule permitting the grain in the first case is based on the legal principle that we assume the *status quo ante* unchanged if there is not sufficient contrary evidence. Since the chances are even that nothing fell into the bin of ordinary grain, our assumption stands. However, in the second case, one bushel did fall into each bin, and therefore one can take a more stringent view since the chances are even that it was the *Terumah* that fell into the ordinary grain, making it forbidden.

7. CONCLUSION

In the framework of Jewish Law, it is natural that probability considerations appear. Because the law requires making manifold decisions every day respecting duties and rights, it deals of necessity with empirical data which are by their very nature often incomplete. The essential question therefore is how convincing are the available data. This accounts for the very many instances of probabilistic reasoning in the Talmud.

In fact some of the rules cited above, and similar ones, originated well before the Talmudic era. On the other hand, most of them have been incorporated into the subsequent codifications of Jewish law up to our own times. However, in practice, their application is often modified by other considerations.

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