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Studies in Babylonian Lunar Theory: Part I. Empirical Elements for Modeling Lunar and Solar Anomalies

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Contents

Introduction

Perhaps the most remarkable and far reaching achievement of Babylonian astronomy was the creation of a unified and comprehensive lunar theory, which combined competent mathematical models of the effects of lunar and solar anomaly on the intervals between syzygies, the variation in the length of daylight, and the motion of the lunar node to predict the instants of syzygies; the occurrences, magnitudes, and directions of eclipses; and the lunar visibilities near syzygies, including the potential visibility of the Moon on the 30th day of any month. Previously, eclipses had been predicted with considerable confidence using the famous 223 month eclipse cycle, familiarly called the "saros",[2] which also underlay clever Goal Year methods for predicting the lengths of months. Mathematically, however, neither of these techniques went much beyond recognizing simple periodicities, modified perhaps by empirical adjustments about which we

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[1] Wilson, WY. This paper and its sequels are dedicated to Asger Aaboe, who securely laid the foundations of a deeper understanding of Babylonian lunar theory and introduced me to these studies. Much of the research for this paper was conducted while I was a Senior Fellow at the Dibner Institute. I am deeply indebted to both the Dibner Institute and Dibner Fund for their generous support. I should also like to thank Dennis Duke, Bernard Goldstein, Alexander Jones and John Steele for their careful readings and helpful comments and suggestions. Naturally, all remaining errors are my own.

[2] For details of the origin and subsequent persistence of this misnomer in 1691 from Edmund Halley's misreading of a passage in Suidas in correcting a textual error in his copy of Pliny, see Neugebauer (1957a), *ad* 51, pp 141–2. In cuneiform texts this interval is simply called "18 years".

can only guess; nor was there any evident effort to unite these two predictive techniques in a common intellectual framework.

This changed profoundly with the invention of the first comprehensive lunar theory, known as System A, which appears to have occurred in the city of Babylon shortly after the beginning of the 4^{th} century B.C. Roughly a century later an alternative theory known as System B was developed, possibly in Uruk, which addressed the same issues with different mathematical schemes, introduced a few improved parameters, and accomplished at least one substantive improvement in the treatment of solar anomaly. In between there is evidence of transitional developments, and subsequently some evidence of further attempts at refinements, but essentially these two versions of lunar theory were the apex of Babylonian scientific astronomy.

It cannot be overemphasized that System A had no analogous precursor, and thus that – unlike virtually all succeeding scientific theories – its invention was a radical departure from anything which preceded it. Consequently, we can say almost nothing with confidence about its motivation, which cannot have been directed at the shortcomings of a (non-existent) prevailing theory, nor seemingly at any urgent practical need, since we have still no evidence of how it was employed. Indeed, how little we know of the circumstances of its origin is reflected in the fact that we do not even know for certain its author's name.

We can, however, infer from the elements of the theory something of the path taken in the course of its assemblage, and in the process gain some appreciation of the intellectual qualities of its author. The crucial step – apart from conceiving the desirability and imagining the feasibility of constructing a comprehensive mathematical model of this complex set of variable phenomena – was the separation and successful modeling of the effects of lunar and solar anomaly on the intervals between syzygies. The crux of this development was the initial construction of a model depicting the variations due to lunar anomaly of the intervals between syzygies. Subsequently, Hipparchus devised a different and more familiar solution, described in the *Almagest*, which first built a solar model from the variation of the seasons, and then a model of the Moon's behavior from residual variations in intervals between eclipses. However, the initial Babylonian solution of this problem proceeded in the opposite order – first modeling the variations due to lunar anomaly, and then building solar models from the amplitudes of the residual variations.

Before proceeding further it may be useful to review the components of the System A theory and their interrelations (Fig. 1). The theory consists of a number of arithmetical functions, typically tabulated in columns, with a logical dependency which moves from left to right. (Here and subsequently I follow Neugebauer's notation in ACT in labeling the separate functions.) The independent variable of the theory is a "month", referenced by name in the first column of ephemerides and treated as a uniform variable, equivalent to syzygy number, despite its actual variation. Full and new moons are treated separately, although with a strictly consistent single theory underlying the two computations. Following the dates come two columns of fundamental arguments. The first, column Φ, governs the variation due to lunar anomaly of intervals between syzygies of the same kind, and is in precise phase with the lunar velocity at syzygy. It turns out to describe the variation due to lunar anomaly of the length of 223 months, constructed so that its

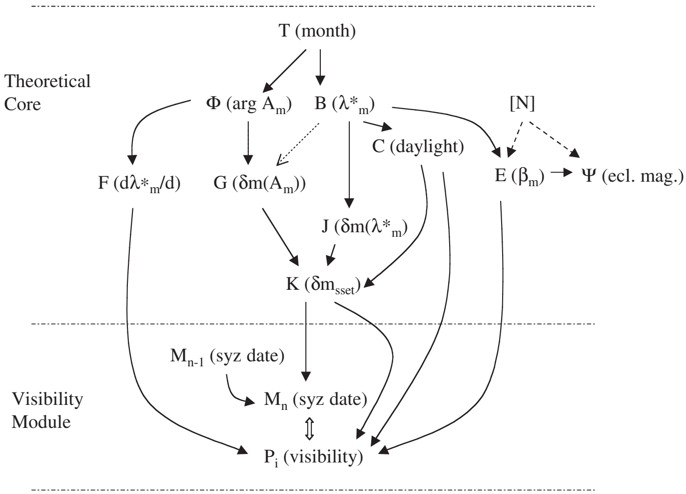

Fig. 1. System A: component functions and their interrelations and dependencies. (After Aaboe, 2001, p. 60)

truncated maximum, 2,13;20($133\frac{1}{3}$) time-degrees,[3] corresponds to the actual maximum of 223 months, disregarding whole days.

The second fundamental argument (column B) is the sidereal longitude of the moon at syzygy, which reflects a variable motion depending strictly on the longitude of the preceding syzygy. This serves as an argument for successive columns describing the length of daylight (column C), the lunar latitude (column E) and the moon's nodal elongation at syzygy (column Ψ), also referred to as "eclipse magnitude". Columns E and Ψ also incorporate, albeit stealthily, a uniform motion of the lunar node, which comprises a third fundamental argument.

The next columns, F and G, return the theory to variations due to lunar anomaly. Column F gives the lunar velocity at syzygy, expressed either as degrees per day or degrees per time degree and, as noted, is precisely in phase with column Φ . Curiously its mean value is closer to the moon's instantaneous mean motion at syzygy, which is increased by the inequality known as the Variation, than it is to the moon's mean motion over longer intervals. Column G gives the variation due to lunar anomaly in the intervals between successive syzygies, constructed so that the maximum of G corresponds to the actual maximum month-length and all corrections for solar anomaly (column J) are subtractive. Its calculation employs column Φ as argument of anomaly, but it also includes a constant derived from the solar model underlying column B.

Column J computes the correction due to solar anomaly as a function of the longitude of syzygy (column B), and the sum of columns G, J, and an adjustment for the

[3] As reflected in several procedure texts, this number was also the name of this function. See ACT, 211-2. One day = 360 time-degrees (^{uš}); the "large hour" (^{H}) = $60^{uš}$, introduced by Neugebauer for typographical convenience had no counterpart in Babylonian usage and should be avoided.

variable sunset epoch derived from column C comprises column K, which finally gives the resulting excess over 29 days of the interval between successive syzygies, measured relative to sunset.

Column K is the first endpoint of the theory, whether used to determine the interval between successive syzygies or, in summation to calculate intervals between eclipses. The steps leading up to its calculation form a logically tight and consistent theoretical core, which can be indefinitely extended. This is followed by a module of columns involving visibility calculations, leading in iterative fashion to the dates (day of the month) and instants of the syzygies (column M) and the intervals between risings and settings of the moon and sun (columns P_i). This visibility module is the final endpoint of the theory and draws upon the longitude (column B) and latitude (column E) of the moon, its daily motion (column F), the interval between syzygies (column K) and a set of coefficients intended to adjust their combined effects for the variable inclination of ecliptic and horizon. All in all it is a strikingly ambitious, clever, and consistent theory built around a mathematical core of uncommon rigor and elegance.

The construction of this theory seems to have broadly followed the order in which functions appear, which also shadows their logical dependencies. The first challenge was separating the effects of lunar and solar anomalies on the intervals between syzygies, which was accomplished by modeling the variation due to lunar anomaly of 223, 12 and 235 months, thereby establishing column Φ, and then by a similar technique modeling the variation of one month. This permitted calculation of the amplitudes [of variation] due to lunar anomaly of 6, 12, and 223 months, and comparison with observed amplitudes from eclipse records established the corresponding amplitudes attributable to solar or positional anomaly. These served as the empirical underpinnings for the subsequent model of solar anomaly, which in turn played a direct role in the functions for length of daylight, lunar latitude, and nodal elongation, and an indirect role (through an additive constant) in the final form of column G. Thereafter, the theory is mostly arithmetic, except for the insertion of new data in the form of prior syzygy dates (M_{n-i}), presumably from eclipse records.

The key to modeling the effects of lunar anomaly was recognizing that the 19-year cycle implies that any anomaly dependent on longitude must vanish over 235 months, making the variation of this interval due solely to lunar anomaly. Since this interval is often bounded by eclipses, its variation by nearly 3/4 of a day is both conspicuous and readily measurable.

A necessary preliminary to the solution was assembling the requisite empirical elements. In addition to two period relations of ancient origin, the "saros" eclipse cycle and "19-year" annual cycle, these included three new elements which were probably the work of the theory's author, to wit

- an accurate anomalistic period relation;
- estimates of the extremes and amplitudes of key eclipse intervals; and
- an improved estimate of the length of the mean synodic month.

The next and crucial theoretical step consisted of building a mathematical model of the amplitude of 235 months in units equal to the change over 223 months due to lunar anomaly in the length of 223 months.[4] Comparing the resulting theoretical amplitude of this interval (235 months) with its observed amplitude measured in time-degrees ($^{\text{uš}}$) defined the magnitude of this unit, while an improved estimate of the mean synodic month anchored the scheme for 235 months in reality and established its extremes in both sets of units. A further estimate of the maximum length of 223 months completed the models for 12 and 223 months and by a similar procedure the model for one month. Finally, aggregations of six individual months gave the variation and resulting amplitude of 6-month intervals, thereby completing the model of variations due to lunar anomaly.

The result was a remarkably successful model of the effects of lunar anomaly on the intervals between syzygies, none of which – except for 235 months – could be measured directly. So successful was this model that System B improves upon it only in its use of a slightly more accurate value for the mean synodic month. Otherwise, and especially in its depiction of the amplitudes of variation of the principal eclipse intervals, the System B theory of lunar anomaly is generally inferior to System A's.

The steps involved in modeling solar anomaly are less clear, but several distinct elements can be identified, to wit:

- refining the period relation implicit in the 19 year cycle;
- inventing a rigorous, yet flexible, method for depicting variations depending on position;
- constructing models of the variation in solar progress consistent with the residual amplitudes of prominent eclipse intervals derived from historical eclipse records and the model of lunar anomaly; and,
- establishing the position of the line of symmetry corresponding to the solar apsidal line.

The first of these steps – refining the annual period relation – was almost certainly accomplished in the course of assembling the requisite empirical elements, if not earlier, and reflected an estimate that the shortfall from complete return in 235 months was $-\frac{1}{2}^{\circ}$, which both lunar theories incorporated in slightly different formulations. The subsequent steps were developed differently in the two theories, with System B improving materially on System A's depiction of variations due to solar anomaly. Between the two there is evidence of a transitional scheme [AB] towards the end of the 4$^{\text{th}}$ century, which replaced the System A solar model with a scheme yielding variations for 6 months similar to that of System B while retaining the System A model for lunar anomaly.

This study examines the issues encountered in the construction of these models in several parts. Part I, which follows, addresses the requisite empirical elements for separating lunar and solar anomalies and their possible derivations. Part II will explore the construction of the System A model for lunar anomaly, by which the separation of the two effects was accomplished, and compare it with the corresponding elements of System B. Part III will first examine the construction of the System A solar model, then its shortcomings, and finally the alternatives evidenced in Systems AB and B. Part IV

[4] It is noteworthy that these are purely abstract units which, apart from issues of accuracy and precision, could not be measured directly, since all changes in the length of 223 months are contaminated by effects of solar anomaly.

will address the construction of the models for latitude and nodal elongation and the possible origins of their underlying period relations.

1. The "Saros" Eclipse Cycle

One of the fundamental tools of Babylonian astronomy was the concept of *period relation* which equated Π phenomena of one sort with Z of another. In lunar theory the first phenomenon was usually months, although sometimes days, while the second could be years (returns in position), or eclipse possibilities (returns in nodal elongation), or cycles of lunar velocity (anomalistic months). In each case the period relation implied that variations associated with the second phenomenon recurred after Π instances of the first phenomenon, and thus that Π phenomena = Z cycles of variation.[5]

Of the two period relations which played critical roles in the theory the first to be recognized was the 223 month eclipse period known today as the "saros", but simply called "18 years" in cuneiform texts. Initially this may have recognized only that

$$223\,\text{months (m)} = 38\,\text{eclipse possibilities (EP)} \cong 6585\tfrac{1}{3}\text{days (d)}, \qquad (1.0)$$

where "eclipse possibility" meant passage of the position of syzygy by either of the lunar nodes. Later, it came to be understood that the saros also reflected an approximate anomalistic period relation, and still later the increment in longitude of the position of syzygy was more precisely defined, so that in addition to its original content, the saros came also to hold that

$$223\ \text{(m)} \cong 239\ \text{anomalistic months}(\text{m}_\text{a}) \cong 18^{\text{rev}} + \text{ca.}10;30^\circ(\Delta\lambda\ \text{syzygy}). \qquad (1.1)$$

In this form the saros implied a number of approximate relationships, to wit:[6]

$$\begin{aligned}
&1\ \text{mean synodic month (m)} = 29;31,50,\underline{18}..\text{days}\\
&1\ \text{anomalistic month}(\text{m}_\text{a}) \cong 27;33;1\underline{3}..\text{days} \cong 0;55,59\,\text{months}\\
&1\ \text{year (y)} \cong 365;15,\underline{35}\,\text{days} = 12;22,7,\underline{50}..\text{months}\\
&1\ \text{eclipse possibility}(\overline{\text{EP}})^7 = 5;52,\underline{6}..\text{months}
\end{aligned} \qquad (1.2)$$

Because it closely approximates return in both position and lunar anomaly, the amplitude of the saros is only 39 time-degrees ($^{\text{uš}}$) or $2\frac{2}{3}$ hours, far smaller than that of

[5] The earliest instance of a period relation involving a cycle of variation is the schematic annual period relation, 12 "months" = 1 year = 360 "days", which forms the basis for the depiction of the annual variation of the length of day and night and the corresponding daily change in lunar visibility described in EAE 14. Early examples of non-schematic annual period relations are found in MUL.APIN (37 months = 3 years), BM 45728 and BM 36731(334 months = 27 years), and BM 36712 (371 months = 30 years). [See Britton (2002[UOS]) for details.]

[6] In the following correspondences underlined numbers are inaccurate.

[7] I use EP to denote the both the months of theoretical eclipse possibilities and the intervals between them, which is unambiguous if the intervals are associated with the months which begin them.

any other short-period eclipse cycle.[8] This made it the handiest and most reliable for estimating whether an eclipse might be seen, and it is difficult to see how the visibility of a prospective eclipse could have been reliably gauged without it.

Origins

It is not known when the saros eclipse cycle was discovered and not impossible that it was recognized in ancient times that after 54 years eclipses of approximately the same magnitude and direction occurred at approximately the same times of day or night, but one month later in a (well tuned) calendar. However, the absence of any hint of this knowledge in either MUL.APIN or the Enuma Anu Enlil (EAE) series, suggests that its discovery was a 1st millennium event, while the confidence exhibited in the eclipse predictions contained in several letters and reports to the Assyrian kings, suggests that it was known to at least some scholars by the middle of the 7th century.[9] Thus it seems most likely to have been discovered sometime after records of eclipses began to be systematically collected in the middle of the 8th century, and probably in the first half of the 7th century, perhaps in response to heightened interest in eclipse predictions and their associated omens in ruling circles.[10]

The simplest path to its discovery would have proceeded from consideration of the intervals between successive visible eclipses and recognition that such intervals were usually a multiple of six months but occasionally, and less frequently, one month less than such a multiple, i.e. either $6n$ or $6n-1$ months where n is some small integer. This implies that successive eclipse possibilities occur on average slightly less often than every six months, and thus that a continuous list of uniformly distributed eclipse possibilities contains intervals of six months interspersed occasionally with intervals of

[8] The 41 month eclipse cycle comprising 7 EP has an amplitude of 0.9 days; all others shorter than 223 months have amplitudes greater than 1.5 days. None but the saros would seem useful for predicting the visibility of specific eclipses, and none entails a close return in position.

[9] E.g. LABS 46 (Balaši), "the eclipse will pass by, it will not occur"; LABS 78 (Nabu-ahhe-eriba), "the eclipse will occur during the morning watch"; ARAK 250 (Nergal-etir), "in month XII day 14 the moon will make an eclipse [whose circumstances will be adverse to the king]; ARAK 251 (Nergal-etir), "in month VI the moon will make an eclipse (concerning) Elam"; ARAK 447 (Tab-silli-Marduk) "there will be no eclipse. I guarantee it seven times, an eclipse will not take place"; ARAK 487 (Nadinu, –648) "[the eclipse] will not pass by, it will occur ..month II ..day 14..morning watch". In addition, the observation by Akkullanu of a solar eclipse in –668 with a magnitude of only 2 digits, related in LABS 148 and ARAK 3, required a careful watch and a firm idea when the eclipse was likely to occur. See also Parpola, LAS 2, p. 51 for a concurring opinion in reference to LABS 46.

[10] The most dramatic example of this interest was the Substitute King ritual, practiced extensively during the reigns of Esarhaddon and Ashurbanipal, in which a substitute "king" (and queen) would be appointed to absorb the unfavorable effects of an adverse eclipse, which would be expunged from the kingdom by their subsequent deaths. Parpola, LAS 2, xxii–xxxii, gives an excellent description of the practice and surveys ancient attestations.

Table 1.1. Lunar eclipses visible in Babylon showing months, M, from 0 Nabonassar, months XII (−746 Feb 6); intervals, dm, between successive eclipses, and cumulative 5-month ("5itu") intervals and EP intervals

JY	Σ M	dm	5itu	Σ (5itu)	Σ EP	JY	Σ M	dm	5itu	Σ (5itu)	Σ EP
-746	0	begin		0	0	-739	94	6		2	16
-746	6	6		0	1	-738	106	12		2	18
-745	12	6		0	2	-735	141	35	5itu	3	24
-745	18	6		0	3	-735	147	6		3	25
-743	**47**	**29**	**5itu**	**1**	**8**	-734	153	6		3	26
-742	53	6		1	9	-732	**176**	**23**	**5itu**	4	**30**
-742	59	6		1	10	-731	188	12		4	32
-741	65	6		1	11	-730	206	18		4	35
-741	71	6		1	12	-727	241	35	5itu	5	41
-739	**88**	**17**	**5itu**	**2**	**15**	-726	247	6		5	42

5 months. Finally, the assumed frequency of such 5-month intervals effectively defines the period relation comprising an eclipse cycle.

How much could be deduced from a simple list of consecutive eclipses and associated month count may be seen from Table 1.1, which shows the eclipses visible in Babylon and the intervals between them for the first few eclipse possibilities beginning with the eclipse in 0 Nabonassar, month XII (−746 Feb 6). Here we find five examples of $6n - 1$ month intervals (labeled "5itu" as in cuneiform texts). The first two reflect intervals of 8EP and 7EP respectively, which immediately excludes a continuing cycle of 47 months with 8 EP. Continuing in this fashion disqualifies a 7 EP (41 month) cycle in −730 (EP 36), a 23 EP (135 month) cycle in −703 (EP 92) and a 22 EP (129 month) cycle in −701 (EP 97).[11] Thus by the beginning of the 7th century the shortest cycles still consistent with the record of visible eclipses were those with either 15 EP (88 months) and two "5itu" or the saros with 38 EP (223 months) and five "5itu".

The 15 EP cycle (alternating groups of 8 and 7 EP) does not develop a conflict until −668 (EP 166) but has an amplitude of 1.6 days, making it of scant use for predicting the visibility of individual eclipses. Furthermore, by −700 the average ratio of EP to 5itu, oscillating around 7.6:1,[12] clearly exceeded $7\frac{1}{2}$:1 leaving the saros cycle as the shortest remaining candidate. Thus by early in the 7th century a simple list of consecutive eclipses and the intervals between them would have sufficed to eliminate all shorter period eclipse cycles, while the utility of the saros, demonstrable in the 7th century from records of eclipses 54 years earlier, would have removed any need to search further. In short, the simple evidence of a (more or less) complete record of eclipses since the middle of the 8th century would have permitted the discovery of the saros by early in the 7th century,

[11] See Appendix B, Fig. 2 for a more extensive tabulation of the intervals between successive eclipses.

[12] E.g. In −703 12 "5itu"(a) ~ 91 EP(n); thus n/a = 7.58. In −699 13 "5itu"(a) ~ 99 EP(n); thus $n/a = 7.62$. In the saros 38 EP(n) = 5 "5itu"(a), hence $n/a = 7.60$. The modern equivalent is very nearly $n/a = 7.623$.

although probably not much earlier without a longer record of historical eclipses than is attested.

Further evidence for recognition of the saros as an eclipse cycle in the 7th century is furnished by the recent and unexpected discovery of records of Lunar Six from as early as −642 and of complete records, implying some calculations, by at least −605 and probably −617. Since the Goal Year method for such computations assumes a detailed understanding of the saros and its properties, this assures that the saros was well known by the latter part of the 7th century. (See Sect. 3, for further details.)

Saros Cycle Arrangements of Eclipse Records

Whatever its precise origins, by early in the 6th century, the saros had already come to serve as the basis for arranging records of historical eclipses. This entailed distributing its 38 EP into groups of 8 or 7 EP separated by 6 months, with 5-month intervals between groups. A single saros would thus contain five groups with 8-7-8-7-8 EP (or some permutation of this order), which turns out to fit very nicely the initial distribution of 5-month intervals reflected in records of lunar eclipses since the beginning of Nabonassar's reign. For convenience I shall call this arrangement Saros Scheme 1 (denoted "SS1"), since other arrangements were devised later to correct the downward drift of eclipses in the SS1 matrix from the inaccuracy of the saros eclipse period. Nevertheless, as J. Steele has shown,[13] this arrangement persisted without conflict for more than 22 saros cycles, an unusually long period, breaking down only with the small (mag 1.2^d) eclipse on −339 Sep 29. in saros cycle 23. (Henceforth "SC" shall refer to a component saros cycle of SS1, "SC*n*" to the n^{th} component SC of SS1, and "EP*q*" to q^{th} eclipse possibility within some SC).

This distribution accompanied by explicit markings of the "5itu" intervals separating each group is already evident in BM 38462, a contemporary compilation of lunar eclipse reports during the reign of Nebuchadnezzar from −603 to −575.[14] Furthermore the long duration of this arrangement without conflict made it especially convenient for compilations of historical eclipse reports. The most extensive of these was an 8-tablet compilation[15] covering all EP for the first 24 saros cycles of SS1 from −746 to −314. This compilation would have encompassed more than all of the eclipse data used in the construction of System A, and probably most, if not all, of that used in System B. Consequently, I have used it as a template for appendices displaying data used in the discussion which follows, recognizing that System A was unlikely to have drawn upon data later than SC 20.

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[13] Steele, (2000), 421–54; see also, Aaboe, *et al* (1991).

[14] Recognized by Steele, (2000, 432). The tablet contains four columns the last of which is blank, suggesting that it was compiled around the time of its latest contents (−575). All five group boundaries are unambiguously implied by the month dates, while at least three (and probably all five in the unbroken tablet) were explicitly marked by the term "5 itu".

[15] See C.B.F. Walker (1997, 17–25). Also J. M. Steele, "Appendix: The Eclipse Texts", in *ADART* V, pp. 390 ff.

Appendix A shows the Babylonian dates of eclipse possibilities for the first 24 saros cycles of SS1. Before the 6th century some of the intercalations are conjectured, as noted, although not their cumulative total. EP at which visible eclipses occur are shaded and highlighted in bold. Appendix B displays calculated data for eclipses visible in Babylon similarly arranged beginning in B1 with Julian years. Here shaded dates reflect the preserved contents of tablets from the 8-tablet compilation, while dates for which reports are preserved are shown in bold, while "5itu" attestations are underlined. Appendix B continues with intervals, dm, between successive eclipses in B2, mean civil times of syzygies (midnight epoch) in B3, eclipse magnitudes in B4, and sidereal longitudes of syzygies in B5. Finally, Appendix C displays the intervals between syzygies exhibited by visible eclipses for 223 months (C1), 235 months (C2), 12 months (C3) and 6 months (C4), all expressed as time-degrees ($^{\text{uš}}$) in excess of whole days.

In short, the saros eclipse cycle, known from at least late in the 7th century and probably earlier, played a dual role in the eventual separation of the effects of lunar and solar anomalies. Empirically, it provided a framework in the arrangement of eclipse records in a matrix of EPs and saros cycles which facilitated, and in some cases enabled, the derivation of parameters critical to the theoretical development. It also provided the crucial theoretical component of this development from recognition that its expression as an approximate anomalistic period relation,

$$223 \text{ months (m)} \cong 239 \text{ anomalistic months}(\text{m}_\text{a}), \tag{1.1a}$$

was equivalent to the precise statement that

$$(223 + \epsilon)\text{m} = (239 + \epsilon)\text{m}_\text{a}, \tag{1.1b}$$

where ε is a small fraction. In this form it served as both the key to separating the two anomalistic effects and as the backbone of the resulting model of lunar anomaly.

2. The 19-year Cycle

The second period relation of relatively ancient origin which played an important role in the development of the theory was the 19-year annual cycle which states that

$$235 \text{ months} = 19 \text{ years} = 254 \text{ sidereal months}, \tag{2.0}$$

implying that

$$1 \text{ year} = 12;22,6,18..\text{months, and} \tag{2.0a}$$

$$\text{d}_{223}\lambda_\text{m} = 10;43..^\circ(\text{mod.}360^\circ) = -\text{d}_{12}\lambda_\text{m}. \tag{2.0b}$$

This further implies that after 235 months syzygies recur at the same place in the sky, something which could be observed from lunar eclipses once adequate records of lunar positions during eclipses had accumulated.[16] Indeed, the mean shortfall in sidereal longitude of the position of the Moon at syzygy after 235 months is only $-0;11^\circ$ ($\pm 0;23^\circ$), which makes the 19-year cycle the most accurate of the short-period alternatives.

The earliest explicit attestation of this period relation is in a text from Uruk giving the schematic dates[17] of the solstices from probably the beginning of the reign of Nabopolassar in −624 through the end of the reign of Cyrus in −529. An earlier text giving the calculated civil dates and instances of all four cardinal phenomena for the reigns of Nabopolassar and Nebuchadnezzar (−625 to −561),[18] employs the 27-year cycle (334 m = 27y) to calculate the schematic dates of the visibilities of Sirius, and another seemingly early text (BM 36712[19]) reflects a 30-year cycle (371m = 30y). These suggest that the superiority of the 19-year cycle had not been recognized by the end of Nebuchadnezzar's reign but probably occurred during the 3rd quarter of the 6th century. By this time the dates of the summer solstices appear to have been known with considerable accuracy,[20] but it seems more likely that the accuracy of this cycle was either directly observed from records of lunar eclipses 235 months apart[21] or deduced from records of Sirius risings spanning 60 years or more.[22] In any event it is clear that the 19-year cycle was familiar by the last part of the 6th century, after which evidence of alternative short period relations disappears.

The 19-year cycle is pretty accurate for tropical phenomena[23] and remained the basis for calculated dates of solstices and equinoxes in subsequent cuneiform astronomical texts. For sidereal phenomena, however, it is less accurate, and at some point prior

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[16] The earliest preserved record of the position of the moon during an eclipse is for the total eclipse on −684 Oct 3 (GN2 = 3917), recorded in BM 35115 (*ADART* V, No. 3) which reports that the eclipse occurred in "mid-Aries", a rough but relatively accurate description since syzygy occurred at sidereal $\lambda^* = 16;20^\circ$, compared with λ^*(mid Aries) $\cong 18^\circ$. The next example, recorded in the same compilation, concerns the eclipse on −631 May 23 (GN2 4568), which reportedly occurred a specified (but broken) distance "behind α-sco". After −572 reports of lunar positions relative to fixed stars appear more frequently, if irregularly. Whether such reports refer consistently to a specific phase is not known.

[17] I.e. dates in "tithis", equal to 1/30th of a month.

[18] BM 36731, published initially in Neugebauer and Sachs (1967) as Atypical Text A, and including additional fragments and commentary in Britton, (2002, Appendix B, 62–70).

[19] Neugebauer and Sachs (1956); see also Britton (2002).

[20] See Britton (2002, Appendix C, 74). For the 19 years from −548 to −530 the schematic dates for SS range from 1 day early (6 instances) to 1 day late (2 instances) averaging 0.2 days early.

[21] Noteworthy are the following near conjunctions of the moon at syzygy with Normal Stars: α-leo in −587 and −549 (470 m); γ-cap in −568 and −549(again); and especially the actual or near occultations during eclipses of θ-oph in −565 and −546 and δ-cnc in −559 and −540.

[22] In 54 years Sirius would rise 2–3 days earlier in the month than initially, while in 60 years it would rise 2–4 days later in the month. The offsetting errors, insensible after a single period but unmistakable after two, would suggest combining the two periods, and in 57 ($27 + 30 = 3 \times 19$) years Sirius would rise on roughly the same day of the month, give or take a day.

[23] In −500 Y(trop) $\cong$ 365;14,33 days, $\overline{\text{m}} \cong 29;31,50,9$ days, and $235\ \text{m} - 19\text{Y}_\text{t} = 0;5$ days, thus requiring 12 cycles or 228 years for the error to accumulate to a whole day.

to the construction of the System A solar model, the small shortfall in longitude was recognized and estimated to equal $-\frac{1}{4}^{\circ}$. By the second half of the 5th century, when the uniform zodiac was being introduced and longitudes of Normal Stars established, sufficient records of lunar positions relative to Normal Stars during eclipses would have accumulated to reflect a barely sensible shortfall from return after 235 months, a sensible excess, perhaps $\cong 1^{\circ}$ (acc. $= 1;16^{\circ}$) after 334 months, and a smaller excess, perhaps $\cong 0;30^{\circ}$ after 804 months, which could have suggested the correction, $\delta\lambda^{*}_{235\mathrm{m}} \cong -0;15^{\circ}$.[24] Alternatively, this may have been simply the single equivalent of a less precise conclusion to the effect that $\delta\lambda^{*}_{235\,\mathrm{m}}$ was "sensible and less than $\frac{1}{2}^{\circ}$".

With this correction the period relation corresponding to an "adjusted (sidereal) 19-year cycle" becomes:

$$235 \text{ months} = \left(19 - \tfrac{1}{1440}\right) \text{ sidereal years} = \left(254 - \tfrac{1}{1440}\right) \text{ sidereal months}, \quad (2.1)$$

implying

$$1 \text{ year} = 12;22,7,56.. \cong 12;22,8 \text{ months [System A]}, \quad (2.1a)$$

$$d_{223}\lambda_{\mathrm{m}} = 10;29..^{\circ} \cong +10;30^{\circ}(\text{mod.}360^{\circ}) \text{ [System B], and} \quad (2.1b)$$

$$d_{12}\lambda_{\mathrm{m}} = -10;44..^{\circ} \cong -10;45^{\circ}. \quad (2.1c)$$

The earliest scheme depicting the longitudes of syzygies is found in a text, seemingly from the second half of the 5th century,[25] containing computed data and observational remarks for 38 solar eclipse possibilities from −474 to −456. The scheme assumes that

$$d_{223}\lambda_{\mathrm{m}} = 10;30^{\circ} = -d_{12}\lambda_{\mathrm{m}}, \quad (2.2)$$

implying that

$$d_{235}\lambda_{\mathrm{m}} = 0;0^{\circ}$$

as in the (unadjusted) 19-year cycle, but also (and inconsistently) that

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[24] Noteworthy examples of conjunctions or near conjunctions at composite intervals of 235 m and 334 m include eclipses of: ζ-tau from −566 to −417; γ-gem from −548 to −445; δ-cnc from −549 to −409; ε-leo from −567 to −456; α-leo from −549 to −419; β-sco from −536 to −452; θ-oph from −565 to −435; and γ-cap from −568 to −484.

[25] The text is preserved in two copies, BM 36599 and BM 36737+, initially published in Aaboe and Sachs (1969) and further discussed as Text S in Britton (1989) and in Aaboe *et al.* (1991, pp. 68–71). The text contains data for solar eclipse possibilities from −474 to −456 corresponding to solar SC 16, and evidently post-dates the introduction of a uniform zodiac. The close agreement of nodal elongations derived from a primitive "magnitude" function unsuited to extrapolation with modern calculations suggests that at least part of the contents originated not long after the latest dates in the text. On the other hand and arguing for a later date is the inclusion of calculated values of columns Φ and F from System A which, as we shall see, are unlikely to have originated before the beginning of the 4th century. On balance it seems likely that most of the text reflected knowledge from the late 5th century, supplemented by calculations from the early 4th century.

$$1\text{ year} = 12;22,7,50..\text{months}, \qquad (2.2a)$$

from the equivalence

$$223\text{ months} = 18 \cdot 360^\circ + 10;30^\circ = 18 + 7/240\text{ years}, \qquad (2.2b)$$

a value closer to that implied by the adjusted 19-year cycle.[26] In System A expression (2.1) was approximated with the period relation, 2783(=Π) months = 225(=Z) sidereal years, which is the shortest close approximation in which Z is a regular number as required by the underlying scheme. In contrast System B appears based on approximations deriving from expression (2.1b). Both, however, rest on the empirical estimate that $\delta\lambda^*_{235\,m} = -\frac{1}{4}^\circ$.

3. Anomalistic Period Relations

A critical prerequisite for separating the effects of lunar and solar anomaly, and unquestionably the most challenging of the empirical elements, was an accurate estimate of the period of the moon's return in speed or, equivalently, of the anomalistic period relation. Unfortunately, we have almost no evidence of how, when, or even whether the issue of lunar anomaly was considered by Babylonian astronomers of the 6th century. While it seems reasonable to assume that the rather sophisticated Goal Year methods for determining lunar visibilities at new and full moon would have been accompanied by an awareness of the moon's variable speed, we have, so far, no evidence to support or refute this assumption. We do know that the variation in the length of a saros and its amplitude was well known in the 6th century, and that at least two different techniques were devised for depicting it.[27] Both, although with different methods, treated the variation as a seasonal one, and thus implicitly as a function of the longitude of syzygy alone. Thus we may confidently infer that by late in the 6th century it was believed that lunar anomaly played no role in the variation of the saros (or, equivalently that the anomalistic month was not distinguished from the sidereal month). However, whether this meant that the saros was assumed to be an accurate anomalistic period or that the issue had not yet been considered, is presently unclear. Thus the early history, if any, of investigations of the moon's variable speed and its effects remains wholly a matter of conjecture. It is only in the 4th century and within the corpus of mathematical astronomy that we encounter

[26] It is noteworthy that despite its extreme simplicity, the scheme depicting the longitudes of syzygies, evidently designed for computational convenience and reflecting no adjustment for solar anomaly, is nonetheless extremely well centered in relation to the Babylonian sidereal zodiac. Modern longitudes of syzygy, adjusted to the Babylonian zodiac assuming a difference of 10.0° in −500, are on average only 0.12° greater than in the text with positive and negative deviations equally distributed. The maximal errors are roughly ±3°.

[27] One, described in TU 11 and recognized by Brack-Bernsen, was modeled on the variable length of the night and thus purely seasonal; the other, recognized in BM 45861 by John Steele (2002), was a zigzag function with an annual periodicity. Both are discussed in detail in a Brack-Bernsen and Steele (2005).

evidence of interest in the issue, by which time the problem had been solved and an accurate anomalistic period derived.

Three anomalistic period relations are found in mathematical astronomical texts. In order of simplicity they are:

$$14 \text{ m} = 15 \text{ m}_a : 1 \text{ m}_a = 0;56,0 \text{ m} \quad (3.0a)$$

$$251 \text{ m} = 269 \text{ m}_a : 1 \text{ m}_a = 0;55,59,6,\underline{28}..\text{m} \quad (3.0b)$$

$$6247 \text{ m} = 6695 \text{ m}_a : 1 \text{ m}_a = 0;55,59,6,\underline{13}..\text{m} \quad (3.0c)$$

$$[\text{modern } (-400) : 1 \text{ m}_a = 0;55,59,6,\underline{34}..\text{m}] \quad (3.0d)$$

The first is a rough approximation, attested only in a relatively late (−318) text (MNB 1856, Neugebauer and Sachs, 1969, Text H), where it appears in an abbreviated function for daily lunar velocity according to System A, based more precisely on the third expression. Nevertheless, it was certainly known much earlier and was very likely the point of departure for determining the others.

The second, which happens to be the most accurate of the three, is known from System B, where it is the basis for Columns F and G. However, it was certainly also known to the author of System A, who seems to have regarded it as a useful approximation of the System A period.

The third is the fundamental period relation encountered in System A, which underlies Columns Φ, F and G. For reasons developed subsequently I think it likely that this period was not only empirically derived but regarded as its author's best estimate of the anomalistic period, with the shorter period being simply a convenient approximation. In any event, both are remarkably accurate descriptions of a phenomenon which is extremely difficult to measure.

Sources of Data

Direct Observations. The chief difficulty encountered in any attempt to derive an accurate anomalistic period relation is that the extremes of lunar velocity are diffuse and poorly defined and therefore quite impossible to establish from relatively crude observations of position with sufficient accuracy to improve materially upon the simple 14 month period relation. Direct observations over the course of only a few months would suffice to show that the Moon's daily progress varied by about 4 degrees per day, approximately in relation to its position, while a careful count of the approximate number of velocity cycles in 235 months – less than 252 by a few days – could have shown that the anomalistic month was slightly longer than the sidereal month (since 235 months ≅ 254 sidereal months) and roughly equal to $27\frac{1}{2}$ days, and thus to a first approximation, 0;56 months as in (3.0a).

However, even a careful examination of observations roughly a century apart, combined with this rough approximation to show the whole number of cycles, would have been hard pressed to reduce the uncertainty in the interval of exact return in velocity to much less than ±2 days, equivalent to an uncertainty of ±10 in the third fractional place in the ratio of m_a/m above,[28] an order of magnitude greater than the errors in the two period relations actually used. Thus, direct observations of passages by reference stars, routinely made after at least −570, cannot reliably have been the origin of such accurate period relations as we find, and some other source must be sought for their derivation.

Eclipse Reports. Reports of lunar and, less importantly, solar eclipses, recorded systematically from the middle of the 8^{th} century onwards, comprise a second source of information about the effects of lunar anomaly, extending over an even longer period than regular direct observations of passages by reference stars. These typically recorded the times that eclipses began relative to sunrise or sunset – later extended to include the times of all 4 contacts – and after about −630 often included rough statements of the Moon's position relative to prominent stars. The earliest reports mention simply the watch ($1/3^{rd}$ of the night) in which the eclipse began, but soon thereafter times were recorded with a precision of $5^{uš}$, which improved after ca. −570 to a precision of $1^{uš}$, without, however, any perceptible improvement in accuracy.[29] From an analysis of all eclipse reports[30] Peter Huber has found these reported times reflect standard errors averaging $\pm 7^{uš}$ (smaller for shorter intervals and conversely), which imply standard errors in measured intervals of roughly $\pm 5^{uš}$ or 20 of our minutes under the most favorable circumstances and twice this amount on average. These are errors characteristic of the measured intervals alone, excluding what cannot have been insignificant errors in reducing these to the intervals between syzygies in the case of eclipses of different magnitudes.

Such eclipse reports, collected and conveniently arranged by early in the 6^{th} century in series separated by 223 months, show clearly that intervals between eclipses exhibit substantial variations, ranging in amplitude from nearly $2\frac{1}{2}$ days ($\sim 823^{uš}$) for eclipses separated by 6 months, to roughly $\frac{3}{4}$ of a day ($\sim 270^{uš}$) for eclipses separated by 12 and 235 months, to only $40^{uš}$ or less than 3 hours for eclipses at sarosly intervals. However, except for 235 months, each of these intervals is contaminated by variations due to solar anomaly, which preclude their usefulness for deriving an anomalistic period relation, absent a model for solar anomaly by which its effects can be removed. Furthermore, even 235 month intervals, whose variation very nearly reflects solely the effect of lunar anomaly, present the same difficulties as direct observations, namely very diffuse extremes, which preclude the precise determinations of phase required for an accurate anomalistic period relation. Since it appears that Babylonian lunar theory first modeled the effects of lunar anomaly and only secondarily those of solar anomaly, we can safely exclude eclipse reports as the source of an accurate anomalistic period relation.

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[28] Calc: Assume for convenience 95 years. 235×5 m $= 1175$ m $\times 15/14 \cong 1259$ m_a + x; x(obs) $\cong +8^d \pm 2^d \cong$ 0;16 m ± 0;4 m.1175 m/1259;20 m_a = 0;55,5<u>8</u>,<u>55</u>.. 1175/1259;12 m = 0;55,59,<u>16</u>,..

[29] J. Steele, (2000b, pp 57–74).

[30] HdM, 28: $\sigma^2 = 2.8^2 + (0.14\, t_{calc})^2$. Evaluated for $t_{calc} = 45^{uš}$ (from sunset or sunrise) gives: $\sigma(45^{uš}) = \pm 6.9^{uš}$, $\sigma(15^\circ) = \pm 3.3^{uš}$, $\sigma(75^{uš}) = \pm 10.8^{uš}$.

Table 3.0. Comparison of saroi 470 months (= 38 years) apart in time degrees (duration of later saros – duration of earlier saros). SC and EP are for earlier saros and according to SS1. SC 1–18 (−746 to −420)

SC	2	3	4	6	9	12	15	17	18
beg	-728	-710	-692	-656	-602	-548	-494	-458	-440
EP	223m	223m	223m	223m	223m	223m	223m	223m	223m
1			**11**						
2						**-11**	**-11**		**-10**
9				**-11**		**-11**			
10						4	2		1
16			**-11**						
17			4		2	1	0		-1
24	2	2		1	0				
25								**10**	
31			-3						
32		8			**10**	**12**	**12**		

However, it is possible and even likely that eclipse reports played a role in the eventual solution by providing evidence that the well known variation in the length of the saros was due in part to lunar anomaly, and thus that the saros was not an exact period of anomaly. As noted, it had long been assumed that this variation was essentially seasonal and thus dependent on the longitude of syzygy which was understood to advance by $10\frac{1}{2}^{\circ}$ in the course of a saros. Thus a logical starting point for any investigation of the anomalistic period would have been to examine whether the variation of the saros was solely a function of position, in which case the saros would indeed be equal to the anomalistic period.

The procedure for such a test would have been simple and straightforward; compare the durations of pairs of saroi 470 months ($d_{470}\lambda^{*} = -0;30^{\circ}$) apart, which would simply involve finding pairs of saroi bounded by eclipses separated by 2SC and 4EP with no intervening 5-month interval. Table 3.0 gives the results for all such intervals which might have been obtained around −400 by an analyst working with modern data. It reflects ample evidence (11 instances) of an amplitude of variation of at least 10 time-degrees[31] among saroi beginning and ending at the same place, which implicitly must be due to lunar anomaly (since the analysis by design excludes positional variations).

Lunar Four. The third source of potential data is the so-called Lunar Four, measures of the times from sunrise (ŠÚ and NA) and sunset (ME and GE_6) to moonset and moonrise on either side of full moon. These are illustrated in Fig. 3.1, and differ in date and sequence depending on whether opposition occurs at night or during the day. As discussed in detail by L. Brack-Bernsen[32] ŠÚ is the rising time of the Moon's negative elongation at sunrise immediately prior to full moon and NA the rising time of its positive elongation 1 day later, and similarly for ME and GE_6 at sunset. Thus the sum of each pair is essentially the rising time of the daily change in lunar elongation ($\Delta\eta$) around opposition, which is sensibly constant over the half day between sunrise and sunset.

[31] A similar comparison of saroi at intervals of 804 months ($d_{804}\lambda^{*} = +0;40^{\circ}$) would encompass more examples and reflect an amplitude due to lunar anomaly of at least $13^{\text{uš}}$.

[32] L. Brack-Bernsen, (1997, 69–79); (1999a, 149–77); (2002, 5–19).

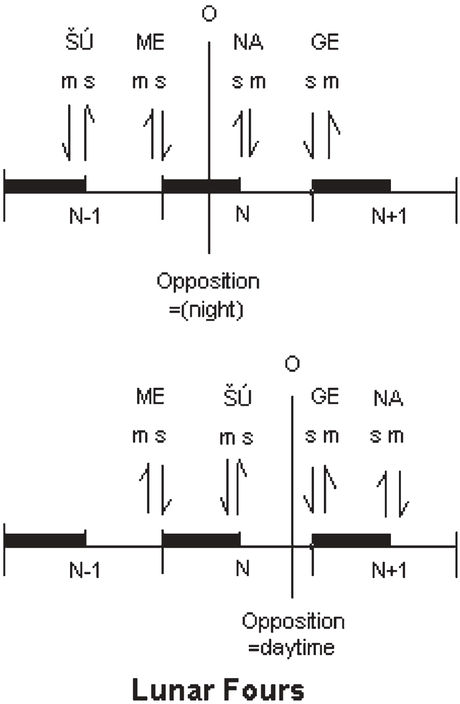

Figure 3.1

This time interval is significantly affected by the angle between the ecliptic and the horizon, which will be as much above or below the equator for ŠÚ and NA as it is below or above the equator for ME and GE_6. As Brack-Bernsen and Schmidt have shown,[33] such effects are very nearly equal in magnitude but in opposite directions at sunrise and sunset, and therefore largely offsetting when the sums of each pair are added together. The resulting quantity, called "Sigma", should therefore closely approximate the right-ascensional component of twice the daily lunar progress in elongation in both magnitude and phase. In fact, several second order effects combine to make Sigma both noisier and slightly larger on average than this description suggests, without, however, introducing any sensible distortion in phase. $\frac{1}{2}$Sigma, therefore, turns out to be not only an excellent proxy for the lunar velocity in phase, but a decent approximation of its average magnitude as well.[34]

Babylonian Goal Year texts, attested from the 3rd century on, regularly record Lunar Four with a fractional precision of 1/6th of a time-degree (uš), month by month for the year 18 years prior to the Goal Year as well as the monthly sums of ŠÚ + NA and ME + GE_6 for that year and the preceding six months. Some of these are accompanied by

33 L Brack-Bernsen and O. Schmidt, (1994, 183–209).

34 For the period −600 to −400, the average value of $\frac{1}{2}$Sigma averaged over whole cycles of lunar anomaly is 13; 4..°/d, compared with an average daily lunar velocity of 13;10,35°/d and an average daily progress in elongation of 12;11,27°/d.

the note "measured", indicating a specific observation, while the completeness of these records and their general agreement with modern computation shows that relatively good methods existed for filling in inevitable gaps in the record due to bad weather or other reasons.

Brack-Bernsen has shown[35] that such inevitable gaps in records of Lunar Four could be and seemingly were filled by a simple "Goal-Year" technique, based on assuming that to a first approximation: (a) 223 months $\cong 6585\frac{1}{3}$ days, and (b) that the intervals ŠÚ+NA and ME+GE were sensibly constant at intervals of a saros,[36] from which it followed that individual Lunar Four could be computed from those one saros (or more) earlier by the formulas,

$$\text{ŠÚ}_{\text{n}} = \text{ŠÚ}_{\text{n}-223} + \tfrac{1}{3}(\text{ŠÚ}_{\text{n}-223} + \text{NA}_{\text{n}-223})$$

$$\text{NA}_{\text{n}} = \text{NA}_{\text{n}-223} - \tfrac{1}{3}(\text{ŠÚ}_{\text{n}-223} + \text{NA}_{\text{n}-223})$$

$$\text{ME}_{\text{n}} = \text{ME}_{\text{n}-223} + \tfrac{1}{3}(\text{ME}_{\text{n}-223} + \text{GE}_{\text{n}-223})$$

$$\text{GE}_{\text{n}} = \text{GE}_{\text{n}-223} - \tfrac{1}{3}(\text{ME}_{\text{n}-223} + \text{GE}_{\text{n}-223}).$$

Thus complete sets of Lunar Four implied that at least some of the entries were calculated which in turn implied the existence of at least fairly complete sets of similar data one saros earlier.

Until recently the earliest evidence of these records was thought to be found in BM 33066, also known as Strm. Cambyses 400 (= LBAT **1477),[37] which reports a complete set of Lunar Six (Lunar Four plus observations of lunar visibilities at the beginning and towards the end of the month) for 13 months beginning with month I of year 7 Cambyses (−522/1). While it has been questioned whether these records reflected contemporary or subsequent observations and calculations,[38] careful examination of the full text and comparisons with modern calculations leaves no appreciable doubt that the text was contemporary with its contents and probably composed in the last two months of 7 Cambyses and in any event before 8 Cambyses, month V.[39]

In December 2005, however, Huber discovered that BM 55554 (ADART V, No. 49), contained complete Lunar Six data for 12 years beginning with 14 Nebuchadnezzar (−590), thereby removing any doubt about the availability of data necessary for Strm. Cambs. 400 and pushing back the certainty of fairly complete Lunar Six data to −608. Shortly afterwards, a text from Nippur (N2349a) was found to contain complete Lunar Six from probably 8 Nabopolassar (−617), and another text (BM 38414) proved to

[35] Brack-Bersen (1997, 1999a), summarized in Brack-Bernsen and Hunger (2002, 34).

[36] Hence implicitly that the saros was an anomalistic period relation.

[37] Initially published by Kugler, SSB 1(1907), 61–75 and most recently by Hunger in ADART V, No 55.

[38] e.g. by Hunger and Pingree (1999, 175) who doubt – erroneously as it turns out – the existence of reliable methods for computing Lunar Six in the 6$^{\text{th}}$ century.

[39] A more complete discussion of the text and this conclusion is contained in the author's paper, "Remarks on Strassmaier Cambyses 400" to appear in a forthcoming festschrift.

contain intermittent Lunar Six data for 5 Kandalanu (−642). Thus we now have firm evidence of Lunar Six data being collected from roughly the middle of the 7^{th} century onwards, and a strong likelihood that relatively complete Lunar Six data were available from around −625 onwards. Finally, from several texts apparently written in the same hand[40] there is evidence of a systematic collection of Lunar Six data from −642 through at least −511, suggesting that someone in the 5^{th} century (or later) was interested in complete records of Lunar Six. Thus by −400 Lunar Four data spanning more than two centuries should have been available for determining the anomalistic period relation.

Theoretical Considerations

Before examining what this data might reveal, some theoretical analysis will help clarify the issue and more precisely define its objective. First, the saros must be close to an anomalistic period, since its amplitude of variation due to lunar anomaly is known to be unusually small. Furthermore since the period relation $223\mathrm{m} \cong 239\mathrm{m_a}$ is equivalent to $(14 \cdot 16 - 1)\ \mathrm{m} \cong (15 \cdot 16 - 1)\ \mathrm{m_a}$,

an accurate anomalistic period relation should be of the form

$$(14 \cdot k - 1)\ \mathrm{m} = (15 \cdot k - 1)\ \mathrm{m_a}, \tag{3.1}$$

where k is some constant and not necessarily an integer. Indeed for the three attested period relations and the saros we have

$$14\ \mathrm{m} = 15\ \mathrm{m_a} : k = \infty$$

$$223\ \mathrm{m} = 239\ \mathrm{m_a} : k = 16$$

$$251\ \mathrm{m} = 269\ \mathrm{m_a} : k = 18$$

$$6427\ \mathrm{m} = 6695\ \mathrm{m_a} : k = 17;55,12\ (17.92),$$

while modern values for m and $\mathrm{m_a}$ circa −400, make $k = 18.04$. Thus the problem of establishing an anomalistic period relation can be redefined as one of finding a suitably accurate value of k.

Now assume that the Moon's daily progress is described by a zigzag function with amplitude Δ and period relation, $\Pi = 14 \cdot k - 1$, $Z = 15 \cdot k - 1$, whence $Z' = Z(\mathrm{mod}\Pi) = k$. The smallest increment of variation of this function will be

$$\delta = 2\Delta/\Pi = 2\Delta/(14 \cdot k - 1),$$

while the monthly increment will be

[40] J.M. Steele, private communication, to be discussed in a forthcoming paper with P.J. Huber.

$$d_1 = 2\Delta \cdot Z'/\Pi = 2\Delta \cdot k/(14 \cdot k - 1) = k \cdot \delta.$$

Finally, the increment for 14 months will be

$$d_{14} = 14 \cdot d_1(\text{mod.}2\Delta = \Pi \cdot \delta)$$
$$= 14 \cdot k \cdot \delta - (14k - 1) \cdot \delta = \delta,$$

so that

$$d_1 = k \cdot d_{14}$$

and thus (3.2)

$$k = d_1/d_{14}.$$

This implies that k can be established if it can be determined how many steps of 14 months are required to make up a month's worth of variation in lunar velocity.

Now if one proceeds in steps (s) of 14 months, it follows from (3.1) that after $q \cdot k$ such steps minus q months the lunar velocity (or some proxy such as Sigma) should be unchanged, i.e. that

$$\Sigma_{n+(14qk-q)m} - \Sigma_n = \Sigma_{n+qk(s)-q(m)} - \Sigma_n = 0. \quad (3.3)$$

However, unless k is an integer, expression (3.3) will not be observed from monthly data, but rather

$$\Sigma_{n+qk'(s)-q(m)} - \Sigma_n = \delta\Sigma,$$

where k$'$ is some integer approximation to k. It follows that if

$$k'_i < k < k'_{i+1},$$

then by linear interpolation,

$$\delta\Sigma(k_i)/(\delta\Sigma(k'_i) - \delta\Sigma(k'_{i+1}) + k'_i = k,$$

and more generally,

$$\delta\Sigma(k'_{i-\alpha}) \cdot (\alpha + \beta)/(\delta\Sigma(k'_{i-\alpha}) - \delta\Sigma(k'_{i+\beta}) + k'_{i-\alpha} = k \quad (3.4)$$

The foregoing are relatively simple consequences of the theory of linear zigzag functions that would have been readily understood by a Babylonian skilled in the use of zigzag functions. They – and especially (3.4) – were, I believe, the point of departure for the determination of the anomalistic period relation.

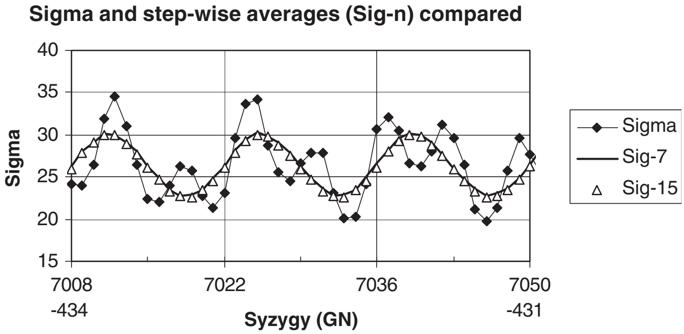

Fig. 3.2. Sigma compared with averages over 7 and 15 14-month steps (Sig-7 and Sig-15), over 3 cycles of lunar anomaly from −434 to −431, an interval chosen to include nearly-extreme values of Sigma

Preliminary Analysis

Let us now examine how this might have been accomplished in practice and with what confidence. First of all the approach evidently requires a reliable proxy for lunar velocity for which monthly data are consistently available over a relatively long interval. The only readily available proxy meeting these conditions would have been Sigma – the sum of the Lunar Four – for which data are now known to exist from at least –642. In what follows I have assumed that complete data were likely to have been available from the accession year of Nabopolassar (−625) and have used Lunar Four computed by a program devised by Peter Huber [LUSIX2], which added together give monthly values for Sigma from −625 on.

Since we shall be interested in the cumulative effects of multiple 14-month increments, it will be useful to arrange the Sigma data in a matrix with 14 columns of successive months, where going from row to row in the same column corresponds to an advance by 14 months. (Values of Sigma used in this analysis and so arranged are given in Appendix D.1 for the years −625 to −565 and −460 to −400). By itself Sigma is quite noisy as illustrated in Fig. 3.2, due largely to the effects of the annual variation in the angle between ecliptic and horizon. However, these effects can be almost entirely eliminated by averaging the Sigma values in a given column of our matrix over a number of 14-month steps (i.e. rows) corresponding to a nearly whole number of years. Since 1 year ≅ 12;22, 6 months, 14 months – 1 year ≅ 1;37,54 months ≅ 0;8 years, whence

$$7 \times 14\text{ m} \cong 0;56\text{ (acc.0;55) years,}$$

$$8 \times 14\text{ m} \cong 1;4\text{ (acc.1;3) years,} \quad \text{and}$$

$$15 \times 14\text{ m} \cong 2;0\text{ (acc.1,59) years.}$$

Consequently averaging Sigma values over 7, 8 or 15 steps should eliminate much of the noise, while averaging over 7 or 15 steps permits the results to be assigned without further complication to the median syzygy. How effective this proves is shown in Fig. 3.2, where both Sig-7 (Sigma averaged over 7 steps) and Sig-15 appear almost perfectly sinusoidal in contrast to Sigma itself. Since the two averages are barely distinguishable,

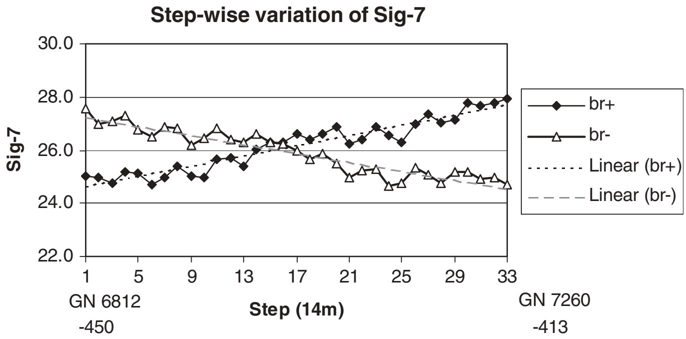

Fig. 3.3. Stepwise variation of Sig-7 over 33 steps from −450 to −413, for values on ascending (br+) and descending (br–)monthly branches

I shall use the simpler Sig-7 in the analysis which follows. (Sig-7 values corresponding to the Sigma values in D.1 are given in Appendix D.2.)

Using Sig-7 now as our proxy for lunar velocity, we shall first test whether 14 months are actually longer than 15 anomalistic months, as previously inferred from the saros. We can do this by tracking the step-wise variation of Sig-7 values on ascending and descending branches of its monthly variation, choosing values near the mean value to best illustrate the trend.

The result is illustrated in Fig. 3.3 which shows values on the ascending branch trending upwards with successive steps and conversely on the descending branch, confirming that the sign in the assumed period relation, $(14k - 1)\mathrm{m} = (15k - 1)\mathrm{m}_{\mathrm{a}}$, is correct.

Next, while we have inferred that the saros, equivalent to k = 16, is not a perfect anomalistic period relation, we need to determine whether 223 months are more or less than 239 anomalistic months and thus whether k is less or more than 16. We can do this in a similar fashion by tracking the sarosly changes in Sig-7 on each branch, again choosing values near the middle of its range to best illustrate any variation. The result is shown in Fig. 3.4 where, in contrast to Fig. 3.3 values decline on the ascending branch at sarosly intervals and conversely on the descending branch. This shows that 223 months are slightly less than 239 anomalistic months, and implies that k must be greater than 16.

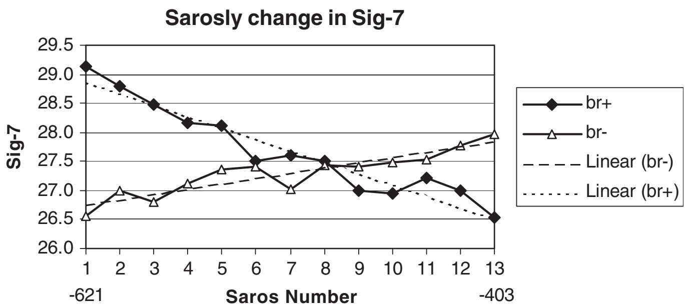

Fig. 3.4. Sarosly variation of Sig-7 over 13 saroi for values on ascending (br+) and descending (br-) monthly branches

Determination of k

To determine k we begin by selecting values of Sig-7 (here denoted by Σ) near the end of the available data on each branch and near the middle of its range. We then compare these with the corresponding values $q \times 14\acute{k}{-}q$ months, where q is as large an integer as the available data allows, and compute

$$\delta\Sigma(k') = \Sigma_{\mathrm{n}} - \Sigma_{\mathrm{n}-14qk'+9}$$

for a range of k'_{i} assumed to include k. Finally, for $k'_{\mathrm{i}} < 0 < k'_{\mathrm{i}+\alpha}$, compute

$$k = \delta\Sigma(k'_{\mathrm{i}}) \times \alpha/\{\delta\Sigma(k'_{\mathrm{i}}) - \delta\Sigma(k'_{\mathrm{i}+\alpha})\} + k'_{\mathrm{i}}. \tag{3.5}$$

The following example illustrates the procedure assuming $q = 9$ ($14q = 126$).

Ascending branch:

	M_n		$M_{n-126k'+9}$		
k'_i	$GN2$	Σ_n	$GN2$	$\Sigma_{n-126k'+9}$	$\delta\Sigma(\mathrm{k}'_{\mathrm{i}})$
16	7386	28.4	5379	29.5	−1.09
17	7386	28.4	5253	29.5	−1.06
18	7386	28.4	5127	28.4	+0.06
19	7386	28.4	5001	27.9	+0.56
20	7386	28.4	4875	27.0	+1.40

Descending branch:

	M_n		$M_{n-126k'+9}$		
k'_i	$GN2$	Σ_n	$GN2$	$\Sigma_{n-126k'+9}$	$\delta\Sigma(k'_i)$
16	7391	26.8	5384	25.5	+1.29
17	7391	26.8	5258	26.2	+0.64
18	7391	26.8	5132	27.0	−0.17
19	7391	26.8	5006	27.6	−0.81
20	7391	26.8	4880	28.8	−1.97

Evidently $\delta\Sigma(\mathrm{k}'_{\mathrm{i}})$ switches sign in the area of $\mathrm{k}' = 18$, and too close to 18 for $\delta\Sigma$ (18) to be a useful datum, so there are 4 possible interpolations using (3.5 above)

	k by linear interpolation from			
	Ascending branch		Descending branch	
with	$\delta\Sigma$ (16)	$\delta\Sigma$ (17)	$\delta\Sigma$ (16)	$\delta\Sigma$ (17)
$\delta\Sigma$ (19)	17.98	18.31	17.84	17.88
$\delta\Sigma$ (20)	17.75	18.29	17.58	17.74
	k(avg) = 17.92			

That this result happens to equal the value of k implicit in the System A period relation is purely coincidental, as may be seen in Table 3.1 Here the same calculation is performed for the same months within a cycle but for each of 13 successively earlier steps as indicated in the column labeled “ds”. The calculation is performed for $q = 9$ and $q = 8$ using four interpolations as above (“4-Ints”) and with only the single interpolation

Table 3.1. *k* computed as above and for successive earlier steps for $q = 9$ and $q = 8$, using 4 interpolations (4-Ints) and 1 interpolation (17v19 Int)

	q=9		q=8	
ds	4-way Int	17v19 Int	4-way Int	17 v 19 Int
0	17.92	18.10	17.74	17.96
-1	17.39	17.62	17.37	17.60
-2	18.03	18.17	18.14	18.40
-3	18.46	18.54	18.47	18.65
-4	17.62	17.81	17.50	17.87
-5	17.88	17.89	17.86	17.92
-6	18.60	18.52	18.49	18.42
-7	17.93	17.97	17.96	17.97
-8	17.98	18.02	17.73	17.53
-9	18.18	18.00	18.15	17.94
-10	17.84	17.66	17.99	18.04
-11	18.28	18.43	17.74	17.57
-12	18.16	18.12	17.84	17.62
-13	17.56	17.35	17.74	17.87
avg	17.99	18.01	17.91	17.95
med	17.96	18.01	17.85	17.93
stdev	**0.34**	**0.34**	**0.32**	**0.34**
σ(μ)	0.09	0.09	0.09	0.09

of δΣ (17) and δΣ (19) ("17v19 Int"). The results are very similar for all 4 examples, reflecting essentially the same standard deviation of ±0.34 for a single calculation, with individual values ranging from 17.35 to 18.65, both within 2σ of 18.04, the actual value of *k* at −500. For $q = 8$ the earliest month in the calculation is in −575, by which time complete Sigma data were unquestionably available.

These results show that a single interpolation between δΣ (17) and δΣ (19) yields as good results (with significantly less calculation) than a four-way interpolation involving δΣ (16) and δΣ (20) as well. Thus to find as good a value of *k* as may be obtained from available data, requires only that we expand the calculation above to the two months (within a cycle) on each branch closest to Sigma's mean value, skip calculations of δΣ (16) and δΣ (20) and interpolate between the resulting averages of δΣ (17) and δΣ (19). This procedure for $q = 9$ yields:

	Branch	
Avg	Ascending	Descending
δΣ (17)	−0.70	+0.91
δΣ (18)	+0.02	+0.10
δΣ (19)	+0.79	−0.76
k(17:19)	17.94	18.09
k(avg)		**18.02**
$\boldsymbol{\sigma(\mu_k)}$		**±0.062**
k(mod. −500)		18.037

The resulting value of *k* is essentially accurate, and reducing the interval covered to 8 k' steps (i.e. letting $q = 8$) has no sensible effect on either the result or its range of uncertainty ($\pm 2\sigma$). Using averages over fewer steps or only one month on each branch expands the 2σ range, but not by much. Observational errors[41] also expand the 2σ range, but apart from systematic errors varying significantly with time, these should largely average out in the process. In short, this relatively simple procedure, applied over 9k'or 8k' steps and employing a single interpolation, yields an excellent value for k, with a 2σ range of uncertainty that probably does not exceed ± 0.2 even after allowing for inevitable errors in the available data. Thus 95% of such analyses should have resulted in determinations that

$$17.85 < k < 18.25, \tag{3.6}$$

a range within which both values from the mathematical lunar theories securely fall. A comparable result could probably have been achieved from a shorter span of data, but only by employing the more computationally demanding 4-way interpolation.

Were the object merely to find the best integer value of k, a much less rigorous procedure applied over a shorter interval would have sufficed to show that

$$17.5 < k < 18.5$$

and thus that $k \cong 18$. However, even a more casual procedure using Sigma values, would have required step-wise averaging to suppress the noise introduced by annual variations of the ecliptic-horizon angle, and once that trouble had been taken, any analyst would have had to be remarkably incurious to not seek a more precise result. That a more precise result was in fact sought is suggested by System A's adoption of

$$k = 17.92(17;55,12),$$

which is otherwise inexplicably precise.[42]

This is not to say that the System A value was uninfluenced by theoretical considerations, such as that it be precisely defined with a terminating sexagesimal fraction. From the sarosly formulation of the period relation,

$$(223 + \varepsilon)\mathrm{m} = (239 + \varepsilon)\mathrm{m_a},$$

41 Huber (2000) analyzes 495 Lunar Four, of which 25% are described as "not observed". The rest exhibit standard errors "modeled fairly well" by $\sigma^{\text{uš}} = \sqrt{[0.6^2 + (0.08t)^2]}$. Data described as "not observed" reflect a distinctively different usage of fractions and standard errors nearly twice as large. Thus the standard error of a typical Lunar Four would have been roughly $\pm 1^{\text{uš}}$, and $\pm 2^{\text{uš}}$ for a single datum of Sigma.

42 In an earlier paper (Britton, 1999), I suggested that the System A period relation, and specifically the substitution of $\varepsilon = \frac{3}{28}$ for $\varepsilon = \frac{1}{9}$ in expression (1.1b), was forced on the author of System A by the relative paucity of discrete values in the 251 month period relation equivalent to $k = 18$. This turns out to be incorrect, for a perfectly good model of the variations due to lunar anomaly can be constructed using the shorter period relation. Thus any attempt to explain the origin of the attested anomalistic period relations needs focus on the more precise value of k found in System A, rather than on the more easily estimated integer value underlying the 251 month period relation.

it can be shown that, k will be precisely defined in this sense, if $\varepsilon = p/q$ where p and q are relatively prime integers and $q - p$ is a regular number. For $p < 10$ and $17.75 < k < 18.25$, there are only five possibilities, to wit:

p	ϵ	Π	Z	k	
1	1/10	2231	2391	17;46,40	(17.77...)
3	3/28	6247	6695	17;55,12	(17.92)
1	1/9	251	269	18	(18.0)
7	7/61	6805	7293	18;4,26,40	(18.074..)
2	2/17	3793	4065	18;8	(18.133..)

Thus a determination that k was between 17.75 and 18 and closer to 18 would fall within the 2σ range and lead naturally to the attested value.

The precise path taken by the author of the period relation undoubtedly differed in some details from that described above. Nonetheless, it seems likely that an approach similar to that described was followed in deriving an accurate anomalistic period relation. One reason (or at least argument) is that both alternative formulations, $k = 17.92$ and $k = 18$, are far more accurate than would be expected from a different method or source of data. Additional evidence is at least suggested by the remarkable similarity of the truncated extremes of System A's column F - depicting the actual extremes of daily lunar velocity at syzygy – and the average extremes for 14-month cycles of $\frac{1}{2}$Sig-7 and $\frac{1}{2}$Sig-15 shown in the brief table below. Both comparisons differ sensibly from the actual extremes of lunar velocity at syzygy as calculated from modern theory, while agreeing closely – and in the case of $\frac{1}{2}$Sig-7 almost precisely – with their counterparts in System A. This seems unlikely to be accidental, and adds (modest) support to the assumption that Sig-7 was indeed the proxy used in determining the period relation.

Lunar Velocity at Syzygy

	System A	**$\frac{1}{2}$Sig-7**	$\frac{1}{2}$Sig-15	Modern
Maximum (M)	**15;0°**	**14;59**	14;57°	15;16°
Minimum (m)	**11;15°**	**11;15**	11;17°	11;49°
Amplitude (Δ)	**3;45°**	**3;46**	3;40°	3;27°

On balance, I think it likely an analysis similar to that outlined above led to the conclusion that k was slightly less than 18, which with ε chosen to yield a precise value of k, led to the System A period relation with $k = 17;55,12$. Whether this analysis was ever repeated remains an open question, but seems at least doubtful. What seems beyond doubt is that the relative accuracy of both periods was no accident,[43] but the result of a well founded analysis of the abundant Lunar Four data.

[43] Goldstein (2002, 6) has observed that rough evidence from Lunar Four, similar to the regression of the extremes of Sigma reflected in Fig. 3.4, could have shown that 14 m > 15 m_a ($k < \infty$), and argued that this plus knowledge that "223 months were a little shorter than 239 anomalistic months"($k > 16$) would allow the anomalistic period relation, 251m = 259 m_a, to be"determined

Table 4.1. Extremes and amplitudes of significant eclipse Intervals; time-degrees ($^{\text{uš}}$) in excess of indicated whole days

	Excess over Whole Days (°)			
	6 M	***12 M***	***223 M***	***235 M***
Whole Days	176	354	6585	6939
Maximum	832	261	134.6	372.6
Minimum	17	−14	95.7	108.3
Amplitude, Δ	815	275	39	264
Δ (moon)	558	248	15	260
Δ (sun)	257	27	24	5

4. Extremes and Amplitudes of Prominent Eclipse Intervals

Another empirical source, which played a critical role in the formulation of the initial theory and may have been the work of its author, was a set of reasonably accurate estimates of the extremes and amplitudes of the intervals between eclipses 6, 12, 223 and 235 months apart. Of these the last two played a crucial role in modeling lunar anomaly, while the amplitude of 6 months, as the largest of the four, should have been the best evidence for modeling solar anomaly from the residuals. The shortest interval of 6 months was also used to anchor the entire System A theory of lunar anomaly to a specific set of syzygies.

Table 4.1 shows the extremes and amplitudes in time-degrees of the relevant intervals between syzygies derived from modern calculations of eclipse times before −400. The largest amplitude, $815^{\text{uš}}$ or $2\,\frac{1}{4}$ days, is for 6 months, which correspond very nearly to half a period of both lunar and solar anomaly. The smallest, $39^{\text{uš}}$ or less than $2\frac{1}{2}$ hours, is for the saros, which closely approximates complete returns in both anomalies. The latter interval, as L. Brack-Bernsen has emphasized,[44] is also the only interval among the four for which the amplitude due to solar anomaly exceeds that due to lunar anomaly.

The best evidence for the length of these intervals and their variations was contained in historical reports of lunar eclipses. As previously noted, except for the earliest reports from the 8th century, these typically gave the interval from eclipse beginning to the nearest of sunset or sunrise expressed in time-degrees with standard errors averaging $7^{\text{uš}}$ in general and half as much for observations within $15^{\text{uš}}$ of sunset or sunrise. Thus apparent

without requiring any additional observations", although elsewhere he remarks on the implicit requirement of additional assumptions. One such assumption would be that $k < 20$, in which case the conclusion, $k = 18$, would have been an obvious choice with an intrinsic uncertainty of ± 1. However, it is hard to see how the relationship, $16 < k < 20$, might have been established with any security except by considerations similar to those illustrated in Fig. 3.4 and 3.5. Furthermore, while taking the analysis no further than this might have supported the integral choice of $k = 18 \pm 1$, it would have provided no grounds for the more precise value of k underlying System A.

44 Initially in Brack-Bernsen (1969, pp 23–28).

intervals between two eclipses of similar magnitude beginning near sunrise or sunset could be expected to exhibit standard errors of roughly $5^{\text{uš}}$ or 20 minutes, while eclipses more distant from sunrise or sunset would typically reflect larger errors. Later reports often included data for all four phases which would have allowed the moment of syzygy to be calculated directly, something especially valuable in comparisons of eclipses of different magnitudes which would require some adjustment for different durations.

In theory Babylonian records of Lunar Four could yield estimates of the moments of syzygies by combining the fractions

$$\text{ME}/(\text{ME} + \text{GE}) \text{ and } \text{ŠÚ}/(\text{ŠÚ} + \text{NA})$$

with appropriate adjustments for their respective evening and morning epochs. In practice, however, the phenomena themselves are too noisy and the recorded observations too imprecise to yield estimates of the moments of syzygy of comparable accuracy to those derived from lunar eclipses.[45] Times of the total phase of solar eclipses might have yielded better estimates of the moments of individual syzygies, but these were too infrequent to have provided evidence of the desired amplitudes. Thus we can safely assume that records of lunar eclipses were the probable source of these amplitudes.

223 Months

The most thoroughly examined of the several intervals was undoubtedly the saros, because of its small variation and prominent role in both eclipse predictions and calculations of Lunar Six. Two apparently early estimates of its amplitude and extremes are attested. One, uncovered by Brack-Bernsen and Hunger in TU 11, a Seleucid text from Uruk[46] recounting what are evidently early procedures, implies that

$$223 \text{ months} = 6585 \text{ days} + 120^{\text{uš}} \pm 20^{\text{uš}} \tag{4.1}$$

and thus that

$$\begin{aligned} &\text{Max}(223\text{m}) - 6585 \text{ days} = 140^{\text{uš}}, \\ &\text{min}(223\text{m}) - 6585 \text{ days} = 100^{\text{uš}}, \text{ and} \\ &\text{Amplitude}, \Delta(223\text{m}) = 40^{\text{uš}}. \end{aligned} \tag{4.1a}$$

The mean is simply the ancient estimate of the saros as $6585\frac{1}{3}$ days, and the variation was assumed to depend on the month and thus solely on solar position. Since solar anomaly contributes the larger part of the variation of the saros, this was not a silly

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[45] Modern calculations of Lunar Four from -540 to -400 yield standard errors in the computed times of single syzygies of $\pm 50^{\text{uš}}$, which compare with standard errors of $\pm 7^{\text{uš}}$ in the reported times of a single lunar phase. These would typically have to be adjusted for differences in magnitude, thus increasing the probable error in the time of a single syzygy, but the resulting uncertainty would remain below $\pm 10^{\text{uš}}$, still a small fraction of the error from Lunar Four. For example, the amplitude of 223 months derived from modern calculations of Lunar Four appears as $72^{\text{uš}}$ ($150^{\text{uš}} - 78^{\text{uš}}$) instead of $39^{\text{uš}}$ ($134^{\text{uš}} - 96^{\text{uš}}$) as it actually is.

[46] TU 11: See especially section VIII.2 pp 80–85.

assumption. Furthermore, the estimated amplitude is remarkable accurate, even though both extremes are too high by $5^{\text{uš}}$.

The second estimate is reflected in a scheme, discovered by J. Steele in the tablet BM 45861,[47] which depicts the variation of 223 months for either successive eclipse possibilities at the same node or at "annual" intervals of some sort. The resulting zigzag function has the parameters

$$\begin{aligned} &\text{Max} = 2{,}20(140)^{\text{uš}} \\ &\text{Min} = 1{,}35(95)^{\text{uš}} \\ &\Delta = 45^{\text{uš}} \\ &\mu = 1{,}57;30\left(117\tfrac{1}{2}\right)^{\text{uš}} \\ &\text{d}_1 = \text{d}_{19} = 5^{\text{uš}} \\ &\Pi = 2\Delta/\text{d}_1 = 18 = P_1. \end{aligned} \tag{4.2}$$

Here the maximum is the same as in TU 11 (too high by $5^{\text{uš}}$) but the minimum has been reduced to an essentially accurate $95^{\text{uš}}$, increasing the amplitude to $45^{\text{uš}}$. The scheme as presented in the text roughly fits the lengths of saroi at intervals of 2EP around the middle of the 7^{th} century, but with the opposite sarosly increment, suggesting that it was simply an arithmetic exercise, which was not intended to replicate the historical record. Nevertheless, it does reflect a new and more accurate value for the mean synodic month of 29;31,50,$\underline{12}$.. days as Steele notes,[48] as well as an essentially accurate estimate of the minimum of the saros.

Appendix C1 shows the lengths of 223-month intervals bounded by eclipses visible in Babylon during the first 24 cycles of SS1. The data are time-degrees ($^{\text{uš}}$) in excess of 6585 days and refer to the interval preceding the EP for which they are entered. For our purposes only the first 20 cycles (through −387) will be considered; the rest are included only to complete the record preserved in the later 8-tablet compilation.

In the first 20 cycles 166 intervals are bounded by eclipses visible in Babylon. For each EP (i.e. in each line) the durations increase or decrease slowly as one moves forward in sarosly increments, frequently passing an extreme for that EP (e.g. 127 for EP 4), which differs materially from the actual maximum or minimum. The actual extremes appear infrequently and occur in neighboring EP. Indeed before −400, each extreme is only clearly evidenced once: the maximum in EP 19(SC 10) bounded by eclipses in −594 and −576; and the minimum in EP 20(SC 10) bounded by eclipses six months later. Both are approached, but not evidently passed by −400 in EPs 11–13 in saros cycles (SC) 18 and 19.

Table 4.2 shows (a) the details for the relevant EP, including the sidereal longitudes of the moon at syzygies near the extremes, and (b) the attested frequency of various interval lengths. Either of two methods (or both) might have been employed to estimate the actual extremes. One method would consider the evident progression, first towards then away from an extreme, and calculate the extreme by linear extrapolation or equivalent estimation. The other would consider focus on the sharp drop in frequency of attested intervals beyond either extreme. Both methods would be subject to observational errors,

[47] Published with commentary in J. Steele (2002, 405–420).
[48] J. Steele, (2002, 417).

Table 4.2. (a) Extremes of 223 months with associated sidereal longitudes, and (b) Frequencies of attested durations

(a)

SC	223m - 6585d (°) EP 18	EP 19	EP 20	λ∗(m)–end EP 19	EP 20
5		131			
6	101		98		
7					
8		134			
9		134		71	
10	98	**135**	**96**	**82**	**256**
11					
12					
13		132	98	104	276
14	101		100		

(b)

Distribution Range >	<	#	Detail Range >	<	#
	95	0		95	0
95	100	9	95	96	1
100	105	14	96	97	0
105	110	15	97	98	2
110	115	26	98	99	5
115	120	18	131	132	8
120	125	36	132	133	4
125	130	21	133	134	2
130	135	27	134	135	3
135		0	135		0

but outlying values would tend to stand out as such. By either method it would have been (just) possible to conclude that

$$\text{Max}(223\text{m}) =< 2{,}15(135)^{\text{uš}} + 6585\,\text{days}, \tag{4.3a}$$

$$\text{min}(223\text{m}) => 1{,}35(95)^{\text{uš}} + 6585\,\text{days, and} \tag{4.3b}$$

$$\Delta(223\text{m}) =< 40^{\text{uš}}, \tag{4.3c}$$

where the quasi-inequality, =</> X, would have been understood as something like “equal to or a little less/more than X”.

The sidereal longitudes shown for the extreme intervals are those for the second eclipse of the pair, calculated by the formula $\lambda* = \lambda + 10^\circ - 1.3825 \times (\text{JY} + 500)/100$ in agreement with the mean longitudes of Babylonian Normal Stars deduced by P.J. Huber (1958).[49] It is noteworthy that the sidereal longitude of the actual maximum in SC 10 occurred curiously near the solar apse of 81;15° found in the proto-System A scheme reflected in BM 36822+ (Aaboe and Sachs, 1969, Text A).

235 Months

In contrast to the saros, no direct estimates of the amplitude of 235 months appear in the cuneiform literature preceding System A. Appendix C2 shows the lengths of 235-month intervals bounded by eclipses visible in Babylon during the first 24 cycles of SS1. Here the data are time-degrees ($^{\text{uš}}$) in excess of whole days, which are mostly 6939 but 6940 for intervals shown as less than $15^{\text{uš}}$. In the first 20 cycles, there are 145 such intervals, slightly fewer than for the saros. However, since solar anomaly has scant sensible effect on this interval, the actual extremes appear more frequently. The maximum

49 Huber’s formulation has been renormed here to −500 when the difference between Babylonian sidereal longitudes and tropical longitudes was $\lambda^* - \lambda_t = 9.98^\circ \cong 10.0^\circ$.

Table 4.3. (a) Extremes of 235 months, and (b) Frequencies of attested durations.

(a)

235m - 6939d (°)									
Min SC	EP 20	Min SC	EP 27	Max SC	EP 14	Max SC	EP 21	Max SC	EP 7
10	113	5	**109**	8	370	2	370	13	365
11		6		9		3	371	14	
12		7	**109**	10		4		15	
13		8		11	**372**	5	**371**	16	370
14	**108**	9		12		6	371	17	
15		10		13		7		18	
16		11		14	371	8	370	19	**372**
17	110	12	115	15		9		20	**372**

(b)

Frequency			Detail		
Range			Range		
>	<	#	>	<	#
	105	0		107	0
105	110	3	107	108	0
110	115	5	108	109	1
115	120	1	109	110	2
360	365	10	370	371	4
365	370	15	371	372	6
370	375	12	372	373	2
375		0	373		0

is approached in all five groups, reached in EP 7 in SC 19 and 20, and clearly passed by in EPs 14 and 21. The minimum is less well attested and approached in EP 27 but clearly passed by only in EP 20.

The relevant data and frequency distribution are summarized in Table 4.3 By either of the methods described above they should have supported a conclusion that

$$370^{\text{uš}} < \text{Max}(235\,\text{m}) - 6939\,\text{days} < 375^{\text{uš}}, \tag{4.4a}$$

$$105^{\text{uš}} < \min(235\,\text{m}) - 6939\,\text{days} < 110^{\text{uš}}, \tag{4.4b}$$

and thus

$$\text{Amplitude, } \Delta(235\,\text{m}) \cong 265^{\text{uš}}, \tag{4.4c}$$

with the precise extremes, probably defined with less confidence than for the more familiar saros with its smaller amplitude.

12 Months

The extremes of 12 months appear to have played no direct role in the construction of the theory, but it is nevertheless instructive to see what the historical eclipse record could have shown. Appendix C3 shows the variation of these intervals for each eclipse possibility in time-degrees in excess of 354 days. In the first 20 cycles there are a total of 140 intervals bounded by eclipses, 5 less (somewhat surprisingly) than for 235 months. The extremes appear only at the beginnings or ends of the series for each relevant EP. Consequently, and unlike the case for 223 and 235 months, the extremes are never clearly passed by in moving from cycle to cycle along an EP, and thus not so precisely defined.

Table 4.4. (a) Extremes of 12 months, and (b) Frequencies of attested durations.

(a)

12m -354d (°)							
Min SC	EP 34	Max SC	EP 33	Max SC	EP 26	Max SC	EP 7
1	beg	1	253	8		15	beg
2	**-10**	2		9	250	16	**258**
3	**-10**	3		10		17	
4		4	**260**	11		18	
5	-7	5	end	12	**258**	19	256
6	-4	6		13	end	20	

(b)

Frequency					
Range			Range		
>	<	#	>	<	#
	-20	0	210	220	5
-20	-10	2	220	230	4
-10	0	6	230	240	13
0	10	11	240	250	7
10	20	12	250	260	6
20	30	9	260		0

The relevant data are summarized in Table 4.4 They suggest only that

$$\text{Max}\ (12\,\text{m}) > 260^{\text{uš}} \tag{4.5a}$$

$$\text{Min}\ (12\,\text{m}) < -10^{\text{uš}} \tag{4.5b}$$

$$\Delta\ (12\text{m}) > 270^{\text{uš}}. \tag{4.5c}$$

Nevertheless it is interesting that intervals shorter than 354 days, appear only in EPs 27 and 34, in the first case only marginally and in the second only in the 7th and 8th centuries when records of eclipse times were less precise than in the 6th and subsequent centuries. Despite this, the author of System A was evidently aware of this phenomenon, since the minimum of 12 months, attested in BM 36311 (discussed in detail in Part II) was accepted as $-20;13,20^{\text{uš}}$.

6 Months

Since 6 months are very nearly half a period of both solar and lunar anomaly, this interval reflects the largest variations from both sources variations, both individually and in combination. As far as we know this interval played no role in the modeling of lunar anomaly, but should have been an important, if not determining element in the subsequent modeling of solar anomaly.

Appendix C4 shows the length of 6 months for each EP over the first 24 saros cycles of SS1, expressed in time-degrees in excess of 176 days. The first 20 cycle displays 172 intervals, 6 more than for the saros. However, extremes are only clearly displayed in EPs 10 and 11, although they appear to be approached in SC 20 of EPs 3 and 4. Given this limited attestation, it may also have been uncertain whether the apparent extremes were indeed close to or inside the actual extremes.

Table 4.5 summarizes the relevant evidence, showing the *average* sidereal longitude of the moon at syzygy, in place of the *ending* longitude in Table 4.2. The best evidence for the extremes is found in EPs 10 and 11, although the actual minimum appears in EP-3, SC-20, without however subsequent confirmation. All together the data suggest that

Table 4.5. (a) Extremes of 6 month intervals bounded by eclipses together with average sidereal lunar longitudes, and (b) Frequencies of attested durations.

(a) / **(b)**

6m -176d (°)											Frequency		
Min	EP	avg	Max	EP	avg		EP	avg	EP	avg	Range		
SC	10	λ∗m	SC	11	λ∗m	SC	3	λ∗m	4	λ∗m	>	<	#
6	59		5	792		14	111		749			10	0
7			6			15					10	20	2
8	28		7			16					20	30	5
9	20	241	8	**832**	45	17	45		801		30	40	2
10			9			18					40	50	4
11	**19**	**262**	10			19					800	810	2
12	25		**11**	**829**	**77**	20	**17**	244	**817**	58	810	820	1
13			12								820	830	1
14			13								830	840	1
15	72		14	781							840		0

$$\text{Max}\ (6\,\text{m}) > 830^{\text{uš}} \tag{4.6a}$$

$$\text{Min}\ (6\,\text{m}) < 20^{\text{uš}} \tag{4.6b}$$

$$\Delta\ (6\,\text{m}) > 810^{\text{uš}}, \tag{4.6c}$$

and place the solar apogee between 64° (SC-20) and 82° (SC-11). The latter number derives from eclipses in −562 for which reports are preserved.[50] Interestingly, the same value for the solar apogee results from the eclipse pair ending in −576 which exhibit the maximal length of the saros, reports of which are also preserved.[51] In both instances, it would have been easy to adopt a somewhat smaller value, which would have agreed better with the actual solar apogee, whose sidereal longitude was roughly 70° at that time. Nevertheless, the closeness of this number with the value 81;15° from the proto-System A scheme in the BM 36822+, which was seemingly rounded to 80° in the completed System A, suggests that these eclipses from the early 6th century played a material role in its determination.

Summary

A close analysis of the historical eclipse record through −400 could have yielded the following estimates for the extremes and amplitudes of the several eclipse intervals and the sidereal longitude of the solar apogee.

50 *ADART* V, No. 7.

51 *ADART* V, No. 6. Obv. II, 1–2 (−594 Nov 27): month VIII. . . it cleared [in the west] and south 2/3 beru ?; Rev III, 10’–12’(−576 Dec 8): [Month IX, the 1–5?, $3\frac{1}{2}$ beru after sunset. It began in the east. All was covered. It cleared in the west. X beru from onset to clearing. [no position data in either].

	223m	*235m*	*12m*	*6m*
	6585^{d}+	*6939*^{d}+	*354*^{d}+	*176*^{d}+
Maximum, *M*	**≅<135**$^{\text{uš}}$	370$^{\text{uš}}$ – 375$^{\text{uš}}$	>260$^{\text{uš}}$	>830$^{\text{uš}}$
Minimum, *m*	≅> 95$^{\text{uš}}$	105$^{\text{uš}}$ – 110$^{\text{uš}}$	< −10$^{\text{uš}}$	< **20**$^{\text{uš}}$
Amplitude, Δ	≅< 40$^{\text{uš}}$	**≅265**$^{\text{uš}}$	>270$^{\text{uš}}$	>810$^{\text{uš}}$
λ*, solar apogee	<82°			<82°

Of these the maximum of 223 months and amplitude of 235 months, shown in bold, played crucial roles in the modeling of lunar anomaly, and it is probably not coincidental that these would have also been among the most securely determinable. Finally, as discussed in the next section, the average of the extremes of the saros seems likely to have played a role in the derivation of an improved value for the mean synodic month.

5. Mean Synodic Month

Apart from phantom variants, abbreviations, and the trivial value, $29\frac{1}{2}$ days, five values for the mean synodic month, $\overline{\text{m}}$, are attested or implied in Babylonian lunar texts. In apparent chronological order these are:

$\overline{\text{m}}$ = 29 days +

		Source
∼190;0$^{\text{uš}}$	(29;31,***40*** ± 0;0,2$^{\text{d}}$)	371 m = 30 × 365;10d (BM 36712), 334 m = 27 × 365;16d (BM36731)
191;1..$^{\text{uš}}$	(29;31,50,***18***..$^{\text{d}}$)	223 m = 65851/3d (ancient "saros")
191;1..$^{\text{uš}}$	(29,31,50,***12***..$^{\text{d}}$)	223 m = 6585d + 1171/2° (BM 45861)
191;0,**36**$^{\text{uš}}$	(29;31, 50, ***6***$^{\text{d}}$)	System A (BM 36311: Text E)
191;0,**50**$^{\text{uš}}$	(29;31,50,***8***,***20***$^{\text{d}}$)	System B (μ(G))
Compare:		
191;0,54..$^{\text{uš}}$	(29;31,50,***9***,***7***$^{\text{d}}$)	Modern (−750 to −350)
191;0,50..$^{\text{uš}}$	(29;31,50,***8***,***21***$^{\text{d}}$)	Modern (−150 to +150)

Omitted from this list is the phantom, rather poor, value, 29;31,50,19,11..$^{\text{d}}$, sometimes described as the System A month-length, which is attested in BM 36705[52] as the result of multiplying the value, m_{a} = 27;33,16,30, by the period relation for lunar anomaly (i.e. × 6695/6247). The origin of this value for the anomalistic month, undoubtedly rounded in its last fractional place, is obscure. Nor is it evident that this arithmetical exercise reflected in BM 36705, was either contemporary with the origin of System A or performed by someone familiar with its construction. What is clear from BM 36311, as shall be discussed in detail in Part 2, is that the System A model for lunar anomaly was based on the more realistic value, $\overline{\text{m}}$ = 29;31,50,6$^{\text{d}}$.

[52] Published in Neugebauer (1957b) with an additional fragment, and (partially) revised transcription in Aaboe (1968, 35–38).

The list reflects increasingly better estimates of $\overline{\mathrm{m}}$, culminating in the famous System B value, which was adopted by Hipparchus, retained by Ptolemy and his Arab successors, and not improved upon until the appearance of the Alfonsine tables in the 13$^{\mathrm{th}}$ century and by Levi ben Gerson in the 14$^{\mathrm{th}}$.[53] This value was slightly, if insensibly, low at the time of its probable derivation around −300, but essentially accurate for the interval between Hipparchus and Ptolemy and for all practical purposes well beyond.

Evidence from estimates of the saros

The earliest estimate, $\overline{\mathrm{m}} \cong 29;31,40^{\mathrm{d}}$, arises in two texts as the month-length implied by explicit year-lengths and associated annual period relations. As such, it may also be a phantom, since one of the texts (BM 36731) was evidently composed after −561, by which time the rough estimate of the saros, $223\ \mathrm{m} = 6585\frac{1}{3}$ d, which was central to Goal Year procedures for computing Lunar Sixes, was probably known. Whatever the case, by the middle of the 6$^{\mathrm{th}}$ century, the known approximation that

$$223\ \text{months} = 6585\tfrac{1}{3}\ \text{days} = 6585^{\mathrm{d}} + 120^{\mathrm{u\check{s}}} \pm 20^{\mathrm{u\check{s}}} \tag{5.1a}$$

implied the improved value, correct to two fractional places of a day, of

$$\overline{\mathrm{m}} = 29^{\mathrm{d}} + 191;1,53^{\mathrm{u\check{s}}} = 29;31,50,\boldsymbol{18}..\text{days}. \tag{5.1b}$$

Subsequently, reducing the minimum of the saros to $6585^{\mathrm{d}} + 95^{\mathrm{u\check{s}}}$ as in the scheme depicting the variation of the saros in BM 45861,[54] lowered the average of the extremes by $2\frac{1}{2}^{\circ}$, implying

$$223\,\overline{\mathrm{m}} = 6585^{\mathrm{d}} + 117\tfrac{1}{2}^{\mathrm{u\check{s}}}, \tag{5.2a}$$

whence

$$\overline{\mathrm{m}} = 29^{\mathrm{d}} + 191;1,12..^{\mathrm{u\check{s}}} = 29;31,50,\boldsymbol{12}..^{\mathrm{d}}. \tag{5.2b}$$

Finally, recognition of essentially accurate values for both extremes of the saros,

$$\mathrm{Max}(223\ \mathrm{m}) = 6585^{\mathrm{d}} + 135^{\mathrm{u\check{s}}};$$

$$\mathrm{min}(223\ \mathrm{m}) = 6585^{\mathrm{d}} + 95^{\mathrm{u\check{s}}},$$

as in (4.3) would have implied that

$$223\,\overline{\mathrm{m}} = 6585^{\mathrm{d}} + 115^{\mathrm{u\check{s}}} = 6585;19,10^{\mathrm{d},55} \tag{5.3a}$$

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[53] See Goldstein, (2003, 65–74) and especially Appendix 3, p. 71 for a survey of attested values in ancient and medieval texts.

[54] As recognized by J. Steele (2002, 417)

[55] C.f. [6585];19,<u>20</u> in ACT 210 and 6585;19,20,58.. from $\overline{\mathrm{m}} = 29;31,50,8,20$.

and thus

$$\overline{\mathrm{m}} = 29^{\mathrm{d}} + 191;0,32..^{\mathrm{uš}} = 29;31,50,\boldsymbol{5}..^{\mathrm{d}}. \quad (5.3b)$$

This unattested estimate, which would round naturally to $191;0,30^{\mathrm{uš}} = 0;31,50,\boldsymbol{5}^{\mathrm{d}}$, is probably the best that could have been reliably achieved from direct measurements of sarosly intervals, and it is hard to imagine that whoever derived accurate estimates of the extremes of the saros would not have made the calculation. It is also hard to believe that anyone engaged with estimating these extremes, would have been unaware of their intrinsic imprecision. For example, a change of $1^{\mathrm{uš}}$ in either extreme, would change $\overline{\mathrm{m}}$ by $\pm 1;20$ in the 3rd fractional place, quite apart from any consideration of the asymmetrical distribution of saros lengths reflected in the historical eclipse record and Table 4.2 Consequently, this result, although an improvement over earlier estimates, could hardly have been thought secure in its third fractional place[56] (of a day).

Evidence from the anomalistic period relation

An alternative, and ultimately more robust, derivation became possible following the derivation of an accurate anomalistic period relation, which made it possible to calculate the interval over which both solar and lunar anomalies most nearly vanish. The calculation would have been trivial, requiring simply the computation of solar progress in various multiples of 251 months as

$$\delta\lambda_{\mathrm{sun}} = (251 \times 19/235) \times 360^{\circ} \times n(\mathrm{mod.}360^{\circ}), \quad (5.4)$$

where n is an integer. Table 5.1 presents the best results for $n \leq 20$ (405y) using the 19-year cycle for mean solar progress. The only interval with a smaller increment of solar progress than the saros is 17 anomalistic periods, and indeed increasing n yields no smaller increment until $n = 109(\delta\lambda = +2^{\circ})$.

If the sidereal version of the 19-year cycle is used in the calculation, $\delta\lambda^*(17)$ becomes $-7;36^{\circ}$,[57] still less than the increment from the saros, which also includes an element of variation from lunar anomaly. Thus by either calculus the single interval which most nearly eliminates the effects of both lunar and solar anomaly is 17 anomalistic periods, equivalent to 4267 months or slightly less than 345 years. Furthermore, since this interval is 30 months or 5(6-month) EP greater than 19 saros cycles, such intervals will frequently be bounded by eclipses at opposite nodes.[58]

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[56] Goldstein (2003, 66–7) recognizes (5.3), and notes that increasing 223 $\overline{\mathrm{m}}$ to $116^{\mathrm{uš}}$ yields $\overline{\mathrm{m}} = 29;31,50,8,\boldsymbol{4}..$ days, which as described in Britton (1999, 198 and 2002, 46) rounds naturally to the equivalent of ..8,20 days. Goldstein suggests that this unattested value might have resulted from assuming that $M(223\mathrm{m}) > 135^{\mathrm{uš}}$, which however, would have been in the wrong direction, making the accuracy of the resulting month-length doubly fortuitous.

[57] Note that this rounds more naturally to $7\frac{1}{2}^{\circ}$, as reported by Ptolemy, than does $-7;45..^{\circ}$ implicit in the System A period relation 2783m = 225y; 1 year = 12;22,8 months.

[58] Since 4267m = 19SC+(30m=5EP), such intervals bounded by eclipses will only occur within EP groups at EPs 1–3 for the earlier of the pair.

Table 5.1. Solar progress, $\delta\lambda$, in $n \times 251$ months

n	months	years	$\delta\lambda$
17	**4267**	**345**	**−3°**
7	1757	142	+20°
10	2510	203	−23°

This interval is, of course, the one famously described in the Almagest (*Alm.* IV.2) as the period relation

$$4267\ \mathrm{m} = 4573\ \mathrm{m_a} = 345\mathrm{y} - 7\tfrac{1}{2}^{\circ} = 126007^{\mathrm{d}}1^{\mathrm{h}}, \quad (5.5)$$

which Ptolemy says Hipparchus investigated, even applying a correction for solar anomaly, in deriving (but actually confirming) 29;31,50,8,20$^{\mathrm{d}}$ as the length of the mean synodic month. Without explicitly saying so, Ptolemy invites the reader to infer that this period relation was Hipparchus's discovery, an invitation accepted by virtually every modern commentator.[59]

In fact there is no more reason to assume that Hipparchus was the author of this interval, than to accept that he fortuitously derived the Babylonian System B value for mean synodic month from it by some arithmetical inadvertence, since as even Ptolemy seems to have recognized[60] along with subsequent commentators,[61] the data cited yields the month length 29;31,50,8,**9**,20.. $^{\mathrm{d}}$, whose 4$^{\mathrm{th}}$ fractional place would round naturally to 10 or 0, but not 20. Far more likely is that this whole period relation was simply part of the System B material, including the mean synodic month and the anomalistic and draconic period relations which somehow came into Hipparchus's possession. Whatever the precise case, the Babylonian author of the anomalistic period relation would almost certainly have done the calculation and been aware that 17 anomalistic periods produced the closest return in both solar and lunar anomaly of any period relation within the limits of the historical eclipse record.

If one accepts that systematic records of lunar eclipses began with the reign of Nabonassar,[62] then the earliest evidence from eclipses 4267 months apart would have come

[59] E.g. A. Aaboe (1955) and G.J.Toomer,(1980) Other commentators, e.g. V. Petersen, *Centaurus*, 11(1966), 306–09 B.L van der Waerden, Centaurus 15 (1970), 21–25); W. Hartner, *Gnomon* 44, 529–37; R Mercier, Archives *Internationales d"Histoire des Sciences,* 27 (1977), 33–71 reflect the same assumption but add nothing of substance to the discussion.

[60] Ptolemy, who certainly performed the calculation, remarks that the division results *εγγιστα* (very nearly) in the value 29; 31, 50, 8, 20.

[61] Goldstein (2003, 68 and 71) and Depuydt (2002, 93) discuss subsequent recognitions of the correct division, beginning with al-Hajjaj in the 9$^{\mathrm{th}}$ century and including (with minor variants) Ibn Yunis (ca. 1000.), al-Biruni (11th c.), Savasorda (1122), al-Bitruji (ca. 1200), I. Israeli (1310), and Copernicus (1543).

[62] Supporting the conclusion that systematic records of eclipses began with the reign of Nabonassar (in addition to Ptolemy's testimony and evidence that the Babylonian Chronicle series also began with this reign) is the arrangement of eclipse records according to SS1. This arrangement

from two eclipse pairs ending in −401 in EPs 6 and 7 of SC 20. These were followed in SC 20 by pairs ending in −398 and −397 (EPs 14 and 15), and in −394 (EP 21), −390 (EP 30), −387 (EP 36) and −386 (EP 38). Thus through −386 and in fact before −379, there were 8 eight instances of eclipse pairs visible in Babylon separated by 4267 months, with initial dates ranging from −746 to −731. If one further accepts that BM 36822 (Aaboe and Sachs, 1969, Text A), which appears to have covered years 6–7 of Artaxerxes 2, was probably composed not long after its last contents (−396), then only the first four of these pairs would have preceded its composition.

The significance of this lies in the fact that notwithstanding various errors and other difficulties, BM 36822 contains values of column Φ, the function which serves as argument of anomaly in the System A model of lunar anomaly, whose construction as evidenced in BM 36311 (Aaboe, 1968, Text E) reflects the mean synodic month,[63]

$$\overline{\mathrm{m}} = 29^{\mathrm{d}} + 191;0,36^{\mathrm{u\check{s}}} = 29;31,50,\underline{\mathbf{6}}^{\mathrm{d}}, \qquad (5.6)$$

a value which with the 19-year cycle also implies the familiar parameter for the mean daily lunar velocity,

$$\mu(d\lambda_{\mathrm{m}}/d) = 254 \times 360^{\circ}/(235 \times \overline{\mathrm{m}}) = 13;10,34,59,30,32..^{\circ}/d \cong 13;10,35^{\circ}/d.$$

Table 5.2 shows temporal data for the four pairs of visible eclipses separated by this interval from −747 through −395. Shown in bold are the shortest times relative to sunset (SS) or sunrise (SR) available for comparison and their increments, d(4267), over the 4267 months. These exhibit increments over whole days ranging (before any adjustment for differences in magnitude) ranging from $10^{\mathrm{u\check{s}}}$ to $27^{\mathrm{u\check{s}}}$. The average interval from sunset or to sunrise was $33^{\mathrm{u\check{s}}}$, corresponding in Huber's analysis to a single standard error of roughly $\pm 5^{\mathrm{u\check{s}}}$ or an interval error of $\pm 7\frac{1}{2}^{\mathrm{u\check{s}}}$. Given complete reports, the smallest interval error should have been from the earliest comparison, both of whose eclipses ended very shortly before sunrise.

However, reports of the first two eclipses - preserved in the tablet BM 41985 (*ADART* V, No.1 = LBAT 1413) - are sparse in detail and very far from complete by the standard of later reports. The first eclipse is simply said to have occurred during the morning watch, which by modern calculation would have begun $67^{\mathrm{u\check{s}}}$ before sunrise. No other temporal detail is preserved, nor does any seem likely to have been included in the compete report. The second report might have originally included the statement that it began in the morning watch[64] – two signs may be missing – but otherwise simply says that the moon set eclipsed which amounts to the same thing. Thus neither report conveys more than that both eclipses began in the morning watch with the second starting somewhat later than the first. Precisely the same circumstances were exhibited by the

would have conflicted with previous visible eclipses in the immediately preceding saros cycle. Had earlier systematic records been available, their structure would have certainly governed the subsequent arrangement, which would have differed from that of SS1.

[63] To be discussed in detail in Part II describing the modeling of lunar anomaly; meanwhile see Britton (1990, 65) and (1999, 219–20).

[64] Began 50° before sunrise.

Table 5.2. Times and magnitudes of eclipses visible in Babylon between –747 and −395 separated by 4267 months. Figures in bold are the shorter, potentially observable intervals and their differences

SC	EP	GN2	Reign	RY	I	M	J Date	(S-,N+) Mag	BMT syz (uš)	beg rel SS (uš)	beg rel SR (uš)	end rel SS (uš)	end rel SR (uš)
1	1	3142	NBNSR	0		XII	-746 Feb 6	11.2	64	134	**-68**	189	**-13**
1	2	3148	NBNSR	1	U	VI	-746 Aug 6	-12.4	55	105	**-46**	155	5
20	6	7409	ARTX2	2	A	XI	-401 Feb 2	8.9	79	149	**-53**	200	**-2**
20	7	7415	ARTX2	3		IV	-401 Jul 29	-11.4	83	131	**-19**	180	30
19	5	4267	change				126007	-2.3	15	15	**14**	11	**10**
19	5	4267	change				126007	1.0	28	26	**27**	25	na
1	9	3189	NBNSR	4		IX	-743 Nov 25	11.6	66	145	**-58**	195	-8
1	10	3195	NBNSR	5		III	-742 May20	-18.0	259	-51	-205	**8**	-146
20	14	7456	ARTX2	6		VIII	-398 Nov 21	12.4	81	158	**-45**	209	6
20	15	7462	ARTX2	7	A	II	-397 May16	-2.9	289	-12	-166	**22**	-132
19	5	4267	change				126007	0.8	15	12	**13**	13	na
19	5	4267	change				126007	15.1	30	na	39	**14**	14

subsequent eclipse of each pair. Both began in the morning watch, and the second set eclipsed. Although they actually occurred $15^{\text{uš}}$ and $28^{\text{uš}}$ later respectively than their earlier counterparts, nothing in the earlier reports would have conveyed or even suggested this. Thus all that might have been confidently assumed from these reports, however detailed or accurate the later reports may have been, would have been that the pairs were separated by very nearly 126007 days.

In the third pair (first of the second set) of eclipses, exactly the same circumstances applied as in the second of the first set. Both eclipses began in the morning watch and the later eclipse set eclipsed. Absent a statement that the earlier eclipse ended before sunrise, no secure inference could be drawn that one preceded the other. The circumstances of the fourth pair differ from the other three in that both began before sunset and ended very shortly after sunset. In fact, the syzygies were separated by an interval of 30° over whole days, but absent more precise data in the earlier report, all that could be inferred with confidence from either pair would have been that the interval in question was again very nearly 126007 days.

The earliest records which report detailed measures of the intervals between eclipse phases and sunset or sunrise are for eclipses in −730 and −712. Both, however are calculations, or at least estimations, since the first eclipse is said to have begun $60^{\text{uš}}$ after sunrise (actually $87^{\text{uš}}$), while the second reportedly began $20^{\text{uš}}$ before sunset (actually $8^{\text{uš}}$ after sunset). Not until −701 do we find evidence of more precise timings in a fragmentary report that the moon "set eclipsed $x^{\text{uš}}$ before sunrise" (actual $= 9^{\text{uš}} <$ SR). In −685 an eclipse is reported $100^{\text{uš}}$ after sunset, which compares reasonably well for such a long interval with the calculated time of $109^{\text{uš}}$. Thus by the beginning of the 7^{th} century the interval timings relative to sunset or sunrise with a precision of $5^{\text{uš}}$ were evidently well established, whereas half a century earlier only the watch of onset and ending is attested. When the change occurred is not known, but it seems likely that the earlier custom persisted for some years. Thus before −395, and perhaps for some while afterwards, a plausible estimate of the length of 4267 months would have been that

$$4267\text{ months} = 4573\text{m}_\text{a} \cong 126007\text{ days},$$

whence

$$\overline{\text{m}} = 29^\text{d} + 191;0,36,16..^{\text{uš}} = 29;31,50,\mathbf{6},\underline{2}..^\text{d}.$$

The value $\overline{\text{m}} \cong 29;31,50,\mathbf{6}^\text{d}$, would been within the uncertainty of the mean month implied by the extremes of the saros (5.3b), with both being uncertain from different sources by roughly ±2 in the third fractional place (of a day). It could also have been a compromise between a slightly larger value derived from the 4267 month interval and a lower value implied by extremes of the saros. Whatever the case, it was the value adopted by the author of System A in constructing the model of lunar anomaly, which seems most easily explained as deriving from the 4267 month interval. This explanation, moreover, places relatively tight time limits on the invention of System A, which cannot have preceded -401 nor occurred too much later, by which time the slight excess of 4267 months over 126007 days would have become evident.

System B's mean synodic month length, 29;31,50,8,20^d

The likely origin of the more famous System B parameter has recently been the object of suggestions by Goldstein[65] and Rawlins,[66] neither of which is supported by any evidence beyond numerical coincidence. In fact, the simplest, and I believe, correct explanation is that the interval cited by Ptolemy,

$$4267\text{ months} = 126007\text{ days} + 15^{\text{uš}} \tag{5.6}$$

was of Babylonian, not Hipparchan, origin and the basis for the System B parameter. As previously described,[67] the latter appears as the mean value of Column G[68] in the form

$$\overline{\text{m}} = 29^\text{d} + \mu(\text{G})^{\text{uš}} = 29\text{d} + 191;0,50^{\text{uš}}. \tag{5.7}$$

[65] Goldstein (2003) suggestion that it arose from estimating the mean duration of the saros as 6585d + $116^{\text{uš}}$ (see note 47 above), implies that its accuracy was the fortuitous result of rounding the mean to whole $^{\text{uš}}$ combined with an adjustment in the wrong direction of the estimated maximum.

[66] Rawlins (2002a, 2002b) has ascribed the parameter to Aristarchus of Samos by an argument which requires changing units, rounding in the middle of a computation, and employs a different year-length from that attributed to Aristarchus.

[67] Britton (1999, 198) and (2002, 46)

[68] Column G is a linear zigzag function which depicts the variation due to lunar anomaly in the length of one month. In System B G's amplitude, $\Delta\text{G}_\text{B} = 156;52,30^{\text{uš}}$, is simply $5/8 \times 60 = 37;30\times$ the amplitude, ΔF_B of Column F, the daily lunar velocity, where $\Delta\text{F}_\text{B} = 4;11^{\circ/\text{d}} = \Pi$ (of the period relation)/60.

In cuneiform numerical notation, units and tens each effectively take up a column of space, so that one column (out of 7) will be saved if the last fractional place of G_B is a multiple of 10, which in turn requires that $\mu(G_B)$ be a multiple of $0;0,5^{uš}$. From (5.7) we get

$$\overline{m} = (126007 \times 360 + 15)^{uš}/4267\,m = 29^d + 191;0,48,56..^{uš}, \qquad (5.8)$$

which rounds naturally to $\mu(G_B)$ as in (5.7). Thus the "20" in the 4^{th} fractional place, which is empirically without meaning, is most simply explained as arising from a desire to save the space otherwise required by an insignificant column of units in the tabulation of Column G. Obviously this explanation only fits a Babylonian context; in a Greek context the parameter would have been left in fractional days and rounded to 10 or 0 (i.e. omitted) in the forth fractional place.

The estimated length of 4267 months as $15^{uš}$ in excess of whole days, would emerge naturally and relatively quickly, once interval timings were recorded for the earlier eclipses of each pair. Table 5.3 shows the lengths of such intervals bounded by eclipses visible in Babylon through −314. For initial eclipses after −730 there are 25 pairs (18 beginning −710 or later), which exhibit extremes of $32^{uš}$ and $10^{uš}$ and an average of $22^{uš}$, in line with the average for this period. Such data reflect the intervals between syzygies, whereas the early eclipse reports would mainly have recorded the times of onset or ending, which would theoretically require some adjustment to make eclipses of unequal magnitude comparable. This fact, together with a precision of $5^{uš}$ in the early reports of eclipse times and the inevitable observational errors would have made $15^{uš}$ a natural estimate of the mean length of this interval with an apparent variation of a similar amount. Extending the period for comparisons would increase the amount of evidence, but not its quality until at least after −200 (i.e. early eclipses after −575), making a better estimate before the creation of System B unlikely, if not impossible.

Table 5.3. Lengths of 4267 months bounded by eclipses visible in Babylon to −314 in time-degrees (^{uš}) over 126007 days. Extremes are highlighted in bold.

	Lunar Eclipses Visible in Babylon: 4267m - 126007d (uš)														
	SC 20			SC 21			SC 22			SC 23			SC 24		
EP	Y1	Y2	uš	Y1	Y2	uš	Y1	Y2	uš	Y1	Y2	uš	Y1	Y2	uš
6	-746	-401	15				-710	-365	19	-692	-347	21			
7	-746	-401	28				-710	-365	25	-692	-347	24			
8															
14	-743	-398	15				-707	-362	15	-689	-344	16	-670	-325	16
15	-742	-397	30	-724	-379	30							-670	-325	29
21	-739	-394	26							-685	-340	29			
22															
23															
24										-684	-339	20	-666	-321	18
29				-717	-372	18	-699	-354	19				-663	-318	22
30	-735	-390	28							-681	-336	24	-663	-318	23
36	-732	-387	31				-696	-351	**32**	-678	-333	32			
37				-714	-369	11	-696	-351	**10**				-660	-315	**10**
38	-731	-386	30	-713	-368	31				-677	-332	32	-659	-314	**32**

In short, it seems that the 4267 month interval cited by Ptolemy, and implicitly attributed to Hipparchus, was indeed the source of the System B mean synodic month-length, but that it originated from a Babylonian rather than Hellenistic analysis that recognized its theoretical significance, rooted in the anomalistic period relations and also estimated its length as $15^{\text{uš}}$ in excess of whole days.

Summary

Five different empirical elements, or sets of elements, went into the separation of lunar and solar anomalies and formed the foundations of the corresponding models in both Babylonian lunar theories. In brief, these were:

1) The "sarosly" period relations concerning attributes of 223 months, first as an eclipse cycle,

$$223\,\text{months} \cong 38\,\text{EP} \cong 242\,\text{draconitic months},$$

by which eclipse reports were arranged in a matrix of 38 EP and 223 month cycles facilitating analytical comparisons, and secondly as an anomalistic period relation, expressed in the form

$$(223 + \varepsilon)\text{ months} = (239 + \varepsilon)\text{anomalistic months},$$

which became the principle scalpel for separating lunar from solar anomaly and formed theoretical backbone of the System A model of lunar anomaly.

2) The 19-year cycle, both in its approximate,

$$235\,\text{months} \cong 19\,\text{years} \cong 254\,\text{sidereal months},$$

and sidereal forms,

$$235\,\text{months} = 19 - \tfrac{1}{1440}\text{sidereal years} = 254 - \tfrac{1}{1440}\text{sidereal months},$$

which underlay the recognition that - uniquely among intervals bounded by eclipses - the variation of 235 months was due solely to lunar anomaly.

3) Accurate anomalistic period relations of the form $14k - 1$ months $= 15k - 1$ anomalistic months,

$$6427\,\text{months} = 6695\,\text{anomalistic months; k} = 17;55,12 (\text{System A})$$

and

$$251\,\text{months} = 269\,\text{anomalistic months; k} = 18 (\text{System B}),$$

whose accuracy, among other evidence, suggests they were derived from analysis of data comprising the Lunar Four.

4) Estimates of the extremes and amplitudes of important eclipse interval depending in considerable part on the sarosly arrangement of eclipse reports, the best inferences from which would have been

Interval	*Excess over*	*Max*	*Min*	*Δ*
223m	6585d	$\leq$ **135**$^{\text{uš}}$	$\geq 95^{\text{uš}}$	$\leq 40^{\text{uš}}$
235m	6939d	$370^{\text{uš}} \sim 375^{\text{uš}}$	$105^{\text{uš}} \sim 110^{\text{uš}}$	$\cong$**265**$^{\text{uš}}$
12m	354d	$>260^{\text{uš}}$	$< -10^{\text{uš}}$	$> 270^{\text{uš}}$
6m	176d	$>830^{\text{uš}}$	$< 20^{\text{uš}+}$	$> 810^{\text{uš}}$

Here the elements in bold appear to have played direct roles in the modeling the effects of lunar anomaly. In addition the extremes of 223 months and 6 months could have suggested that the sidereal longitude of the solar apse was roughly 82° or less, without, however, offering much evidence of how much less, if any.

5) Improved estimates of the length of the mean synodic month derived from estimates of the length of 4267 months as 126007 days plus an increment of

$$\text{System A: } +\ 0^{\text{uš}}, \text{ whence } \overline{\text{m}} = 29^{\text{d}} + 191;0,36^{\text{uš}} (= 29;31,50,6^{\text{d}})$$

and

$$\text{System B: } +\ 15^{\text{uš}}, \text{ whence } \overline{\text{m}} = 29\text{d} + 191;0,48,56..^{\text{uš}} (= 29;31,50,8,9..^{\text{d}})$$
$$\cong 29^{\text{d}} + 191;0,50^{\text{uš}} (= 29;31,50,8,20^{\text{d}}).$$

The first would be tenable only so long as the times of early eclipses were reported in terms of their watch, implying that System A must have been constructed in the first half of the 4$^{\text{th}}$ century.[69] Conversely, the second would have required more precise estimates of the times of eclipse phases, implying that it was derived after the middle of the 4$^{\text{th}}$ century. Both would have been consistent with the uncertainty of the estimated extremes and amplitudes of 223 months, the System B parameter implying a slightly higher minimum than 95$^{\text{uš}}$.

Omitted here is any discussion of improved period relations describing the intervals between eclipses, which played a determinative role in the schemes depicting lunar latitudes and eclipse magnitudes in both theories. These involved additional issues whose examination would have substantially prolonged the preceding discussion. Since they played no role in separating the effects of lunar and solar anomaly, they have been left for a separate discussion.

In tracing the possible development of each of these parameters I have tried to show how each might have been derived by relevant analysis applied to empirical material known to be available at the times in question. In the case of the anomalistic period relation – the most challenging of the five and the only derivation in which considerable averaging must have played a role – the fact that a regular integer value of *k* turned out to be almost precisely accurate was certainly fortuitous, but understanding that the actual value must be very close to this could hardly have been so. Moreover, the alternative value of *k* underlying the System A period relation, seems strong evidence of a robust derivation of this parameter.

[69] A fuller discussion of the evidence bearing on the chronologies of Systems A and B will appear in Part III. However, the balance of evidence suggests that System A was created near the beginning of the 4th century B.C and System B perhaps a century later.

The estimates of the extremes and amplitudes of prominent eclipse intervals depended critically on the organization of eclipse reports in sarosly groupings. Finally the improved, and essentially accurate value of the mean synodic month encountered in System B, seems to have been due partly to a greater abundance of attested 4267 month intervals but also to a greater precision in the reports of the earlier eclipses defining such intervals. None of these results supports the notion that these parameters arose from simple averages or numerical manipulations of rough observations. Instead, each appears to reflect a rational and sometimes inspired use of the extensive, if not very accurate, observational material which we know to have been available.

Abbreviations

ACT	O. Neugebauer, *Astronomical Cuneiform Texts*, 3 vols., (London, 1955), Lund Humphries. Reprinted (1983), Springer-Verlag.
AACD	*Ancient Astronomy and Celestial Divination*, ed. N.M. Swerdlow, (Cambridge, MA and London, 1999), MIT Press.
AfO	*Archiv für Orientforschung*, Verlag Ferdinand Berger & Sönne G.M.B.H.
AHES	*Archive for History of the Exact Sciences*, Springer-Verlag.
Alm.	*Ptolemy's ALMAGEST*, translated and annotated by G.J. Toomer, (New York, Berlin, Heidelberg, Tokyo, 1984), Springer-Verlag.
AOAT	*Alter Orient und Altes Testament*, Ugarit-Verlag, Munster
ARAK	H. Hunger, "Astrological Reports to Assyrian Kings", SAA 8 (1992)
ADART	
I-III	A.J. Sachs and H. Hunger, *Astronomical Diaries and Related Texts from Babylonia,* Vol. I (–651 to –261), 1988; Vol. II (–260 to –164), 1989: Vol. III (–163 to end), 1995 (Vienna, Verlag d. Österreich. Akad. d. Wiss.)
V	H. Hunger, *Astronomical Diaries and Related Texts from Babylonia: Lunar and Planetary Texts,* including materials by A.J. Sachs with an Appendix by J.M. Steele, Vol. V, 2001 (Vienna, Verlag d. Österreich. Akad. d. Wiss.)
FsAaboe	*From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences presented to Asger Aaboe,* ed. by J.L. Berggren and B.R. Goldstein, (Copenhagen, 1987), Univ. Library
HAMA	O. Neugebauer, *A History of Ancient Mathematical Astronomy,* (New York - Heidelberg - Berlin, 1975), Springer-Verlag.
HdM:	P.J. Huber and S. de Meis, *Babylonian Eclipse Observations from 750 BC to 1 BC,* (2004), IsIAO – Mimesis
HdO1	*Handbook of Oriental Studies,* Abt.1, The Near and Middle East
JCS	*Journal of Cuneiform Studies* (New Haven, Cambridge MA, Philadelphia, Ann Arbor).
JHA	*Journal for the History of Astronomy*, Science History Publications, UK
KDVSMM	*Kongelige Danske Videnskabernes Selskab, Matematisk-fysiske Meddelelser* (Copenhagen).
LABS	S. Parpola, *Letters from Assyrian and Babylonian Scholars,* SAA 10 (1993)
LAS	S. Parpola, Letters from Assyrian Scholars to the Kings Esarhaddon and Assurbanipal, Part I: Texts (1970); Part II Commentary and Appendices (AOAT, 1983), Neukirchener Verlag
MUL.APIN	See Hunger – Pingree (1989)

SAA	*State Archives of Assyria* (Neo-Assyrian Text Corpus Project – Academy of Finland), Helsinki University Press
TU 11	Brack-Berenson, L. and Hunger, H., "TU 11: A collection of Rules for the Prediction of Lunar Phases and of Month Lengths", *SCIAMVS*, 3 (2002), 3–90
UOS	*Under One Sky*, A. Inhausen and J.M. Steele, eds., Papers delivered at a Symposium held at the British Museum 25–27 June, 2001, AOAT, 297 (2002)

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Appendices

The figures which follow present selected information concerning lunar eclipse possibilities, lunar eclipses visible in Babylon, and intervals between the latter arranged by eclipse possibilities (EP) in saros cycles (SC) according to the scheme (SS1) reflected in several compilations of early eclipse records and especially a compilation comprising 8 tablets covering the 24 saros cycles from the beginning of the reign of Nabonassar to −314.

A. Eclipse Possibilities

A.1. Babylonian dates of lunar eclipse possibilities from 0 Nabonassar, month XII (-746 Feb 6), to 2 Alexander 4, month II (-314 May 29). Before Nabopolassar, attested intercalations are designated by capital A(XII2) or U(VI2), conjectured intercalations in lower case letters

A1-1. Dates of Eclipse Possibilities (EP) arranged in Saros Cycles (SC) according to Saros Scheme 1 (SS1)

beg> EP	-746 SC 1		-738 SC 2		-710 SC 3		-692 SC 4		-674 SC 5		-656 SC 6		-638 SC 7		-620 SC 8		-602 SC 9		-584 SC 10		-566 SC 11		-548 SC 12		<beg SS1 EP
1	**NBNSR**	**XII**		XII		**XII**		**XII2**	6 a	XII2	11 A	I	**9**	**I**	**5 U**	**II**	2 U	II	20	II	38	II	7	II	1
2	**1 U**	**VI**	3 u	VI	**11**	**VI**	**NLUSZ**	**VI**		VI		**VII**		**VII**		VII		**VII**		**VIII**		**VIII**		**VIII**	2
3		**XI**		XI		**XII**		**XII**		XII		**XII2**	**10 A**	**I**	6	I	3	I	**21 U**	**II**	**39**	**II**	8	II	3
4	**2**	**V**	**PULU**	**V**	12 u	VI	**MUSMK**	**VI**	**7**	**VI**	12	VI		**VII**		**VII**		VII		**VII**		**VIII**		VIII	4
5		XI		**XI**		**XI**		XII		**XII**		**XII**		XII2	7 A	I	**4**	**I**	22	I	40	II	**9**	**II**	5
6	3 U	V	2 a	V	SARG2	V	**2 u**	**VI**	8 A'	VI	13	VI	**11**	**VI**		VII		VII		**VII**		**VIII**		VIII	6
7		X		XI		XI		XI		XII		XII		XII		XII2	5 U	I	23 A	I	41 U	II	10 U	II	7
8	4	IV	ULULA	IV	2 a	V	3	V	9	V	14 A	VI	12	VI	8	VI		VI2		VII		VII		VII	8
9		**IX**		IX		**X**		**X**		X		**XI**		XI		**XI**		**XI**		XII		**XII**		**XII**	9
10	**5**	**III**	**2**	**III**	3	III	4 a	IV	**10**	**IV**	**15**	**IV**	13 A	V	**9**	**V**	**6**	**V**	24	V	**42 A**	**VI**	**11**	**VI**	10
11		**IX**		**IX**		IX		**X**		**X**		X		**XI**		**XI**		XI		**XI**		**XII**		XII	11
12	**6 u**	**III**	3 a	III	**4**	**III**	**SANH2**	**III**	11 U	IV	16	IV	**14**	**IV**	10 U	V	7 U	V	**25**	**V**	43	V	12 A	VI	12
13		**VIII**		**IX**		IX		**IX**		**IX**		X		**X**		**X**		X		**XI**		**XI**		XII	13
14	7	II	4	II	5 a	III	2	III	**12**	**III**	17 U'	IV	15	IV	**11**	**IV**	8	IV	26 A	V	**AMLMK**	**V**	13	V	14
15		VIII		VIII		IX		IX		IX		IX		X		X		X		XI		XI		XI	15
16	**8 a**	**I**	5	I	**SANH1**	**I**	3 U	II	13	II	18	II	**16 a**	**III**	12 A	III	9 U	III	27	III	2 A	IV	14	IV	16
17		**VII**		VII		**VII**		VII		VIII		**VIII**		IX		**IX**		**VIII**		IX		**X**		**X**	17
18		XII2	**MKAID**	**I**	**2**	**I**	4	I	**SSSUK**	**II**	**19**	**II**	17	II	13	II	**10**	**II**	**28 A**	**III**	NLSUS	III	15 A	IV	18
19	**9**	**VI**	**u**	**VI2**		VII		**VII**	**U**	**VII**		VIII		**VIII**		**VIII**		**VIII**		**IX**		IX		**X**	19
20		XII		**XII**	**BELIB**	**I**	5 A'	I	**2**	**I**	**20 u**	**II**	18	II	14	II	**11 A**	**II**	**29**	**II**	2	III	**16**	**III**	20
21	10	VI	**2**	**VI**		**VII**		VII		**VII**		**VII**		VIII		**VIII**		VIII		**VIII**		**IX**		IX	21
22		XII		XII		XII2		XII2	3 A	I	KANDL	I	19 U	II	15 U	II	12	I	30	II	3 A	III	**17**	**III**	22
23	11 u	VI	3 a	VI	2	VI	6	VI		VII		VII		VII		VII		VII		VIII		IX		IX	23
24		X		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII**		**XII**	31 U	I	4	I	**CYRUS**	**II**	24
25	**12**	**IV**	**4**	**IV**	3 a	V	**7**	**V**	**4**	**V**	2 a*	VI	**20**	**VI**	**16**	**VI**	13	VI		VI2		**VII**		VIII	25
26		**X**		**X**		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII**		**XII**	NBNID	I	**2 U**	**II**	26
27	**13 a**	**IV**	5	IV	ASNSM	IV	**8 a**	**V**	**5**	**V**	3	V	**21**	**VI**	**17**	**VI**	14 A	VI	32	VI	**A**	**VII**		**VII**	27
28		X		**X**		**X**		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII**		**XII2**	3 A	I	28
29	14	III	6 a	IV	2	IV	**ASRHD**	**IV**	6 A	V	4	V	**22 U**	**VI**	**18 U**	**VI**	15	V	**33 A**	**VI**	**2**	**VI**		VII	29
30		IX		X		X		X		XI		XI		XI		XI		XI		XII		XII		XII2	30
31	**NBNDZ**	**II**	7	II	**3 A**	**III**	**2**	**III**	7	III	5 U	IV	NBPLS	IV	19	IV	16	IV	34	IV	3 A	V	4	V	31
32		VIII		**VIII**		**IX**		IX		**IX**		**IX**		X		**X**		**X**		X		**XI**		**XI**	32
33	**2 a**	**II**	**8**	**II**	4	II	**3 A**	**III**	**8**	**III**	6	III	**2 A**	**IV**	**20 A**	**IV**	17 A	IV	35	IV	**4**	**IV**	5	V	33
34		VIII		**VIII**		**VIII**		IX		**IX**		**IX**		X		**X**		**X**		X		**X**		**XI**	34
35	UKNZR	I	**9 A**	**II**	**5**	**II**	4	II	**9 U**	**III**	**7**	**III**	3	III	21	III	**18**	**III**	**36 A**	**IV**	5	IV	**6 A**	**V**	35
36		**VII**		**VIII**		VIII		**VIII**		**VIII**		IX		**IX**		**IX**		IX		**X**		**X**		XI	36
37	2	I	10	I	6 a	II	5 a	II	10	II	8 U	III	4	III	NBKDR	III	19	III	37	III	**6 A**	**IV**	**7**	**IV**	37
38		VII		VII		VIII		VIII		VIII		VIII		IX		IX		IX		IX		X		X	38

U/u, A/a = attested/conjectured VI2, XII2; A',U' = attested in Assyria; a* = XII2 conjectured, year known; A(U) = both attested, A likely; A! = U expected
Eclipses at least partially visible in Babylon are shaded and in bold; scheme breaks down with marginally visible eclipses after 6 months in EP 24, SC 23 and 24

A1-2. Dates of Eclipse Possibilities (EP) arranged in Saros Cycles (SC) according to Saros Scheme 1 (SS1)

beg> SS1 EP	-530 SC 13		-512 SC 14		-494 SC 15		-476 SC 16		-458 SC 17		-440 SC 18		-422 SC 19		-404 SC 20		-386 SC 21		-368 SC 22		-350 SC 23		-332 SC 24		<beg SS1 EP
1	8	III	9	III	27 A	IV	9	IV	6	IV	24 A	V	DARI2	V	19	V		VI		VI		VI2		VII	1
2		IX		**IX**		**X**		X		**X**		**XI**		XI		**XI**		**XII**		XII		**XII**	4 U	I	2
3	**9 U**	**III**	**10**	**III**	28	III	10 A	IV	**7**	**IV**	25	IV	2 A	V	**ARTX2**	**V**	19	V	37 A	VI	9	VI		VI2	3
4		**VIII**		**IX**		IX		**X**		**X**		X		**XI**		**XI**		XI		**XII**		**XII**		XII	4
5	CAMBS	II	11 U	III	**29**	**III**	**11**	**III**	8 a	IV	**26**	**IV**	**3**	**IV**	2 A	V	**20 (?)**	**V**	**38**	**V**	10 A	VI	**5**	**VI**	5
6		**VIII**		**VIII**		IX		**IX**		**X**		X		**X**		**XI**		XI		**XI**		**XII**		XII	6
7	**2**	**II**	12	II	30 U	III	**12 A**	**III**	9	III	27 A	IV	**4**	**IV**	**3**	**IV**	21 A	V	**39**	**V**	**11**	**V**	ALEXG	VI	7
8		VIII		VIII		VIII		IX		IX		X		X		X		XI		XI		XI	A	XII	8
9	3 U	I	13 A	I	**31**	**I**	13	I	10 A	II	28	II	5 A	III	4	III	22	III	40 A	IV	12	IV	2	IV	9
10		VI2		**VII**		**VII**		VII		**VIII**		**VIII**		IX		**IX**		IX		**X**		**X**		X	10
11		XII		**XII2**	**32 A**	**I**	14	I	**11**	**I**	**29 A**	**II**	6	II	5 A	III	**23**	**III**	**41**	**III**	13 A	IV	3	IV	11
12	**4**	**VI**	14	VI		**VII**		**VII**		VII		**VIII**		**VIII**		IX		**IX**		**IX**		**X**		**X**	12
13		**XII**		**XII**		**XII2**	15 A	I	**12**	**I**	**30**	**I**	7 A	II	6	II	**24 A**	**III**	**42**	**III**	14	III	**4 A**	**IV**	13
14	5 A	VI	**15**	**VI**	33	VI		**VII**		**VII**		VII		**VIII**		**VIII**		IX		**IX**		**IX**		**X**	14
15		XII		XII		XII		XII2	13 A	I	31	I	8	I	**7 A**	**II**	**25**	**II**	43 A	III	15	III	**5**	**III**	15
16	6	IV	16 A	V	34	V	16	V		VI		VI		VI		VII		VII		VIII		VIII		VIII	16
17		X		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII2**	26 A	I	44	I	16 A	II	6	II	17
18	**7**	**IV**	17	IV	35 A	V	**17**	**V**	**14**	**V**	32 A	VI	9	VI	8	VI		VII		VII		VIII		VIII	18
19		**X**		X		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII2**	**45 A**	**I**	17	I	**7 A**	**II**	19
20	**8 A**	**IV**	**18**	**IV**	36	IV	**18 A**	**V**	**15**	**V**	33	V	**10 A**	**VI**	**9**	**VI**	27	VI		**VII**		**VII**		VIII	20
21		**X**		**X**		X		**XI**		**XI**		XI		**XII**		**XII**		XII		**XII2**	**18 A**	**I**	PILIP	I	21
22	**DARI1**	**III**	19 U	IV	**XERXS**	**IV**	**19**	**IV**	16 A	V	34	V	**11**	**V**	10 A	VI	28	VI	**46**	**VI**		VII		**VII**	22
23		IX		IX		X		X		XI		**XI**		XI		XII		XII		XII		XII2	2 A	I	23
24	2	II	20	II	2 U	III	20	III	17	III	35 A	IV	12	IV	11	IV	29 A	V	ARTX3	V	*19*	*VI*		*VII*	24
25		VIII		**VIII**		VIII		**IX**		**IX**		X		X		**X**		XI		XI		XI		XII	25
26	**3 U**	**II**	21	II	3	II	**21 U**	**III**	18	III	36	III	**13 A**	**IV**	**12**	**IV**	30	IV	**2 A**	**V**	**20**	**V**	3	V	26
27		VII		**VIII**		**VIII**		**VIII**		**IX**		IX		**X**		**X**		X		**XI**		**XI**		XI	27
28	4	I	**22 A!**	**II**	4 A	II	ARTX1	II	**19 A!**	**III**	**37**	**III**	14	III	**13 A**	**IV**	**31**	**IV**	3	IV	21 A	V	**4**	**V**	28
29		**VII**		**VIII**		VIII		**VIII**		**IX**		**IX**		**IX**		X		**X**		**X**		XI		**XI**	29
30	5 A	I	23	I	5	I	2 A	II	**20**	**II**	38 A!	III	**15**	**III**	**14**	**III**	32 A	IV	4	IV	**ARSES**	**IV**	**5 A**	**V**	30
31		VI		VI		VI		VII		VII		VIII		VIII		VIII		IX		IX		IX		X	31
32		XII		**XII**		**XII**		XII2	21 A	I	39	I	16 A!(U	II	15	II	33	II	5 A	III	2	III	6	III	32
33	6	V	**24 A**	**VI**	6	VI	**3**	**VI**		**VII**		VII		VIII		**VIII**		VIII		**IX**		**IX**		IX	33
34		XI		**XII**		**XII**		XII		**XII2**	**40 A**	**I**	**17**	**I**	16 U	II	**34**	**II**	6	II	DARI3	III	**7**	**III**	34
35	**7**	**V**	25	V	**7 a**	**VI**	**4**	**VI**	22	VI		**VII**		**VII**		VII		**VIII**		**VIII**	A	IX		**IX**	35
36		**XI**		**XI**		XII		**XII**		**XII**		XII2	18 A	I	**17**	**I**	35 U	II	**7**	**II**	**2**	**II**	ALEX4	III	36
37	8 A	V	**26**	**V**	**8 (?)**	**V**	5 A	VI	23	VI	**41**	**VI**		**VII**		VII		**VII**		**VIII**		VIII	**A**	**IX**	37
38		XI		XI		XI		XII		XII		XII		XII2	**18 A**	**I**	**36**	**I**	8 U	II	**3**	**II**	**2**	**II**	38

U/u, A/a = attested/conjectured VI2, XII2; A',U' = attested in Assyria; a* = XII2 conjectured, year known; A(U) = both attested, A likely; A! = U expected
Eclipses at least partially visible in Babylon are shaded and in bold; scheme breaks down with marginally visible eclipses after 6 months in EP 24, SC 23 and 24

A.1. Abbreviations of Babylonian royal names used in A1, with variant spellings and reigns

A2. Babylonian Royal Names

Abbreviation	Common Name	Ptolemy	Babylonian Name	Written	Reign
Assyrian Hegemony					
NBNSR	Nabonassar	Nabonassar	Nabû-nāsir[1]	^{md}AG-PAB [/ŠEŠ-*ir*]	14 (-746 : -733)
NBNDZ	Nadin	Nadi	[Nabû-]nādin[-zēri][2]	^{md}AG-*na-din*-NUMUN[3]	2 (-732 : -731)
			[Nabû-]šuma-ukin[4] 2	^{m}AG-MU-GIN[5]/GI.NA	0 (-731)
UKNZR	Ukinzer	Kinzer	[Nabû-]mukin-zēri[6]	^{md}AG-GIN-NUMUN	3 (-730 : -728)
PULU	Tiglath-Pileser 3	Por	Tukulti-apil-éšárra[7]	$^{m\ giš}$TUKUL-*ti*-A-É.ŠÁR.RA, ^{m}*Pu-lu*	2 (-727 : -726)
ULULA	Shalmaneser 5	Ilulai	Šulmanu-ašared[8]	^{md}Šul-man-a-šá-red[9], ^{m}Ú-lu-la-a-a	5 (-725 : -721)
MKAID	Merodach-Baladan 2	Mardokempad	Marduk-apla-iddina 2	^{d}AMAR.UTU-A-MU	12 (-720: - 709)
SARG2	Sargon 2	Arkean	Šarru-kîn	^{m} MAN-GI.NA	5 (-708 : -704)
SANH1	Sennacherib	(interregnum)	Sin-ahhe-riba	^{md}30-PAB^{meš}-SU	2 (-703 : -702)
			Marduk-zakir-šumi 2		0 (-702)
MKAID	Merodach-Baladan 2	Mardokempad	Marduk-apla-iddina 2	^{d}AMAR.UTU-A-MU	0 (-702)
BELIB	Belibni	Belib	Bel-ibni	^{md}EN-ib-ni	3 (-701 : -699)
ASNSM	Aššur-nadin-šumi	Aparanad	Aššur-nadin-šumi	^{d}AN.SAR-SUM-MU	6 (-698 : -693)
NLUSZ	Nergal-ušezib	Regebel	Nergal-ušezib	^{d}U.GUR-KAR	1 (-692)
MUSMK	Mušezib-Marduk	Mesesemordak	Mušezib-Marduk	^{m}Mu-še-zib-^{d}AMAR.UTU	4 (-691 : -688)
SANH2	Sennacherib	(interregnum)	Sin-ahhe-riba	^{m}30-PAB^{meš}-SU	8 (-687 : -680)
ASRHD	Esarhaddon	Asaridin	Aššur-aha-iddina	^{m}AN.SÁR-ŠEŠ-SUM-na	12 (-679 : -668)
SSSUK	Šamaš-šum-ukin	Saosdouchin	Šamaš-šuma-ukin	^{md}GIŠ.NU-MU-GI.NA	10 (-66<u>6</u> : -647)
KANDL	Kandalanu	Kiniladan	Kandalanu	^{m}Kan-da-la-nu	21 (-646 : -626)
		(Assyrian interregnum)			1 (-625)
Neo-Babylonian					
NBPLS	Nabopolassar	Nabopolassar	Nabu-apla-usur	^{md}AG-A-PAB	21 (-624 : -604)
NBKDR	Nebuchadnezzar 2	Nabokolassar	Nabu-kudurri-usur 2	^{md}AG-NÍG.GUB-PAB/ú-su-ur	43 (-603 : -561)
AMLMK	Evil-Merodach	Illoaroudam	Amel-Marduk		2 (-560 : -559)
NLSUS	Neriglissar	Nerigasolassar	Nergal-šarra-usur	^{md}U.GUR-MAN$^{?}$-PAB	4 (-558 : -555)
			Labaši-Marduk		0 (-555)
NBNID	Nabonidus	Nabonadi	Nabu-na'id	^{md}AG-na-id	17 (-554 : -538)
Achaemenid					
CYRUS	Cyrus 2	Cyrus	Ku-raš	^{m} Ku/Kur-raš	9 (-537 : -529)
CAMBS	Cambyses 2	Kambyses	Kambyzia	^{m}Kám-bu-zi-ia	8 (-529 : -521)
			Bardiya		0 (-521)
			Nabu-kudurri-usur 3		0 (-521)
			Nabu-kudurri-usur 4		0 (-520)
DARI1	Darius 1	Darius 1	Dariamus 1	^{m}Da-ri-a-mus	36 (-520 : -485)
XERXS	Xerxes	Xerxes			21 (-484 : -464)
			Bel-šimanni		0 (-481)

A.2. Continued

			Šamaš-eriba		0 (-481)
ARTX1	Artaxerxes 1	Artaxerxes 1	Artakšatsu 1	^{m}Ar-tak-šat-su	41 (-463 : -423)
DARI2	Darius 2	Darius 2	Dariamus 2	^{m}Da-ri-a-mus	19 (-422 : -404)
ARTX2	Artaxerxes 2	Artaxerxes 2	Artakšatsu 2	^{m}Ar-tak-šat-su [Memnon]	46 (-403 : -358)
ARTX3	Artaxerxes 3	Ochus	Artakšatsu 3	^{m}Ar-tak-šat-su, ^{m}Ú-ma-su	21 (-357 : -337)
ARSES	Arses	Arogos	Arses	^{m}	2 (-336 : -335)
DARI3	Darius 3	Darius 3	Dariamus 3	^{m}Da-ri-a-mus	5 (-334 : -330)
Hellenistic					
ALEXG	Alexander 3(Great)	Alexander of Macedon	Aliksandar	^{m}A-lik-sa-an-dar	8 (-329 : -322)
PILIP	Philip Arrhidaeus	Philip	Philip	^{m}Pi-líp-su, ^{m}Pi	7 (-321 : -315)
ALEX4	Alexander 4	Alexander 2	Aliksandar	^{m}A-lik-sa-an-dar	4 (-314 : -311)
S.E.	Seleucid Era	(Ptolemaic dynasty)		^{m}Si-lu-ku, si-i	170 (-310 : -141)
A.E.	Arsacid Era			^{m}Ar-šá-ka-a	140+(-140 :)

1 Nabu is the protector.
2 Nabu is the giver of offspring
3 KL A,\; Na-din/di-nu (Bab Chr)
4 Nabu establishes the name (i.e. the legitimate line)
5 KL A, BabChr; GI.NA (CT 34, 46).
6 Nabu is the establisher of the seed (i.e. gives legitimate offspring)
7 My trust is in the son of Ešarra (i.e Aššur).
8 Šulmanu is preeminent.
9 Bab Chr; also Šùl-man/ma-nu-BAR (various KL); Ululaia = born in Ululu

B. Lunar Eclipses Visible in Babylon, Arranged by Saros Cycle According to SS1

B.1. Julian years

B1. Lunar Eclipses Visible in Babylon: Julian Years

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	JY	EP
1	**-746**		-710	-692			-638	-620											—						1
2	**-746**		-710	-692		-656	-638		**-602**	-584	-566	-548		-512	-493		-457	**-439**		-403	-385		-349		2
3	**-745**		-709	-691		-655	-637			-583	-565		-529	-511			-457			-403	-385				3
4	**-745**	-727		-691	-673		-637	-619		-583	-565		-529	-511		-475	-456		**-420**	-402		**-366**	-348		4
5		-726	-708		-672	-654		-618	**-600**			-546			-492	-474		-438	-420		-384	**-366**		**-330**	5
6				-690			-636	-618		-582	-564		**-528**	-510		-474	-456		-419	-401		-365	-347		6
7													-527			-473			-419	-401		-365	-347		7
8																									8
9	-743		-707	-689	-670	-652	-634	-616	**-598**		**-562**	-544	—		-490										9
10	-742	-724			-670	-652		-616	-598		**-562**	-544		-508	-490		-454	-436		-400		**-363**	**-345**		10
11	-742	-724		-688	-670		-633	-615		-579	**-561**			-507	-489		-453	-435			**-381**	**-363**			11
12	-741		-705	-687			-633			**-579**			**-525**		-489	-471		-435	-417		**-381**	**-363**	-344	**-326**	12
13	-741	-723		-687	-669		-632	-614		**-578**	-560		-524	-506	-488		-452	-434			-380	**-362**		**-326**	13
14					-668			-614			-560			-506		-470	-452		-416	-398		**-362**	-344	**-325**	14
15																				-397	-379			-325	15
16	-739		-703	**-685**	—	—	**-631**			—															16
17	-739		-703			**-649**		-613	-595		-559	-540		-504	-486		-450	-432		**-396**					17
18		-720	-702		**-666**	-648			-594	**-576**			**-522**	-504		-468	-450								18
19	-738	-720		**-684**	**-666**		-630	-612	**-594**	**-576**		-540	**-521**		-485	-467		-431	**-413**		-377	**-359**		-323	19
20		-719	**-701**		**-665**	-647			**-593**	**-575**		-539	-521	-503		-467	-449		-413	**-395**		-359	-341		20
21		-719	-701		-665	-647		**-611**		-575	-557		-521	-502		-466	-448		-412	-394		-358	-340		21
22												-538	-520		-484	-466			-412			-358		-322	22
23																									23
24		-717	-699		-663	-645		-609	**-591**			-537											*-339*	*-321*	24
25	-735	-717		-681	-663		-627	**-609**			-555			-501		-465	-447			-392	-374				25
26	-735	-716	-698	-680		-644	-626		**-590**	**-572**		**-536**	-518			**-464**			-410	-392		-356	-338		26
27	-734			-680	-662		-626	**-608**			**-554**	**-536**	-518	**-500**	**-482**	**-464**	**-446**	-428	-409	-391		-355	-337		27
28		-716	-697	-679	-661		-625	-607		**-571**	**-553**			-499			-445	-427		-391	-373			-319	28
29				-679			-625	-607		-571	-553		-517	-499		-463	-445	-427	**-409**		-372	-354		-318	29
30																	-444		**-408**	-390			-336	-318	30
31	-732		-696	-678																					31
32		-714	-696		-660	-642		-605	**-587**		-551	-533		-497	-479										32
33	-731	-713		-677	-659		-623	-605			-551			-497		-461	-443		**-407**	-389		-353	-335		33
34		-713	-695		-659	-641		-605	**-586**		-550	-532		-496	-478		-442	**-424**		-388	**-370**			**-316**	34
35		**-712**	**-694**		**-658**	-640			-586	-568		-532	-514		-478	-460		**-424**	**-406**		**-370**	**-352**		**-316**	35
36	-730	-712		-676	-658		-622	-604		-567	-549		-513	-495		-459	**-441**		**-405**	-387		-351	**-333**		36
37											-549	-531		-495	-477			**-423**	**-405**		-369	-351		-315	37
38																				-386	-368		-332	-314	38

bold = report exists; shaded = texts 2,3,4 (Diaries V) of 8-tablet compilaton; underlined = "5itu" designation

B.2. Intervals in months, dm, between successive visible eclipses

B2. Lunar Eclipses Visible in Babylon: Interval, **dm**, in Months since Prior Visible Eclipse

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	dm	EP
1	beg		**17**	**23**			**23**	**17**																	1
2	6		6	6		23	6		23	29	23	17		23	17		23	23		17	**11**		17		2
3	6		6	6		6	6			6	6		23	6			6			6	6				3
4	6	35		6	35		6	18		6	6		6	6		29	6		29	6		23	12		4
5		6	12		6	12		6	18			18			18	6		18	6		12	6		29	5
6				12			12	6		12	12		12	12		6	12		6	12		6	12		6
7													6			6			6	6		6	6		7
8																									8
9	**29**		**23**	**17**	**23**	**23**	**17**	**17**	**23**		**17**	**23**			**23**										9
10	6	29			6	6		6	6		6	6		23	6		23	29		17		17	17		10
11	6	6		12	6		12	6		29	6			6	6		6	6			35	6			11
12	6		18	6			6			6			29		6	29		6	29		6	6	12	41	12
13	6	12		6	12		6	12		6	12		6	12	6		12	6			6	6		6	13
14					6			6			6			6		12	6		12	24		6	12	6	14
15																				6	12			6	15
16	**17**		**23**	**17**			**17**																		16
17	6		6			41		17	41		17	41		17	23		17	23		**11**					17
18		29	6		23	6			6	29			29	6		23	6								18
19	12	6		18	6		18	12	6	6		12	6		12	6		12	29		23	29		23	19
20		6	12		6	12			6	6		6	6	12		6	12		6	18		6	35		20
21		6	6		6	6		12		6	24		6	6		6	6		6	6		6	6		21
22												12	6		18	6			6			6		18	22
23																									23
24		**17**	**17**		**17**	**17**		**17**	**23**			**11**											18	12	24
25	35	6		35	6		35	6			23			23		17	23			23	35				25
26	6	6	12	6		12	6		12	29		12	23			6			23	6		23	**11**		26
27	6			6	12		6	12			12	6	6	12	29	6	12	47	6	6		6	6		27
28		12	12	6	6		6	6		12	6			6			6	6		6	18			23	28
29				6			6	6		6	6		12	6		12	6	6	12		6	12		6	29
30																	6		6	12			18	6	30
31	**23**		**17**	**11**																					31
32		23	6		23	35		17	35		17	29		17	29										32
33	12	6		12	6		23	6			6			6		23	17		17	17		23	17		33
34		6	12		6	12		6	12		6	12		6	12		6	29		6	29			23	34
35		6	6		6	6			6	35		6	35		6	12		6	12		6	12		6	35
36	18	6		18	6		18	12		6	12		6	12		6	12		6	12		6	18		36
37											6	12		6	12			12	6		12	6		12	37
38																				12	6		12	6	38

B.3. Times of syzygies in Babylonian mean hours from midnight (BMT)

B3. Lunar Eclipses Visible in Babylon: Babylonian Mean Time of Syzygy (hours from midnight)

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	BMT	EP
1	4.3		18.9	1.9			22.3	4.9																	1
2	3.6		19.6	3.8		20.5	5.1		22.4	7.2	16.0	0.9		18.5	3.2		20.3	4.6		20.8	4.7		19.9		2
3	4.7		20.0	3.5		18.1	1.2			22.1	5.0		18.9	1.8			23.3			21.9	5.8				3
4	18.3	1.8		17.2	1.0		17.0	1.2		17.9	2.3		19.3	3.7		20.5	4.7		20.6	4.4		19.4	2.6		4
5		19.3	3.5		19.5	3.3		18.6	2.1			0.3			22.4	5.8		21.0	4.7		20.5	4.7		21.3	5
6				23.9			22.1	5.8		21.4	5.4		21.6	5.8		21.9	5.9		21.6	5.3		20.1	3.3		6
7													0.6			23.1			21.8	5.5		21.3	5.4		7
8																									8
9	4.4		22.1	6.9	15.5	0.1	8.6	16.9	1.0		16.8	0.4			22.5										9
10	17.3	0.1			20.9	4.1		18.8	2.3		18.0	2.1		18.7	3.3		20.6	5.4		23.0		16.6	1.2		10
11	15.7	0.2		17.3	1.7		18.2	2.2		17.8	1.3			22.8	5.7		19.0	1.6			21.0	3.5			11
12	4.2		18.8	2.1			0.5			23.9			0.5		17.6	2.4		20.1	5.0		23.0	7.9	16.8	1.5	12
13	19.8	4.0		20.3	4.5		20.5	4.3		19.5	2.9		17.2	0.1	6.8		20.0	2.5			21.8	4.2		17.4	13
14					2.4			1.1			0.6			1.3		18.3	3.0		20.5	5.4		23.1	7.9	16.6	14
15																				19.3	2.1			22.8	15
16	4.6		19.7	3.0			0.5																		16
17	6.4		22.9			0.6		18.0	2.8		20.2	4.9		21.8	6.2		22.4	6.4		21.9					17
18		21.7	4.8		18.6	1.3			21.1	3.6			23.4	6.2		20.0	3.2								18
19	19.4	3.7		20.8	5.5		23.2	8.1	17.1	2.0		19.8	4.6		21.9	6.4		22.9	6.9		22.7	6.4		21.5	19
20		24.0	6.9		20.4	3.0			22.2	4.6		17.5	0.0	6.6		20.3	3.4		18.1	1.7		17.4	1.5		20
21		20.1	4.5		21.5	6.2		23.9		17.7	2.6		20.2	4.9		22.0	6.3		22.4	6.3		21.5	4.9		21
22												21.2	4.1		18.1	1.3			24.0			0.1		16.9	22
23																									23
24		17.2	1.6		18.0	2.1		17.8	1.5			0.1											22.1	6.7	24
25	23.2	5.9		19.6	2.6		17.1	0.6			23.7			23.8		16.0	0.0			23.7	7.3				25
26	22.9	7.7	16.4	1.0		17.9	2.1		18.1	2.0		17.3	0.9			23.3			21.7	5.3		20.7	4.6		26
27	0.3			20.1	2.9		17.1	0.4			23.4	7.4	15.5	23.7	7.9	16.2	0.6	8.8	17.1	1.3		17.3	1.2		27
28		23.6	8.3	17.0	1.5		18.2	2.3		18.0	1.6			23.8			21.3	4.4		18.6	1.8			23.9	28
29				23.4			21.2	4.8		20.4	4.5		21.2	5.7		23.0	7.7	16.4	1.1		18.4	2.8		19.5	29
30																	5.3		18.5	1.1			21.3	4.2	30
31	3.7		18.5	1.9																					31
32		22.2	6.3		22.6	6.6		22.6	6.3		21.5	4.9		19.1	2.0										32
33	20.6	4.2		19.2	2.6		17.6	1.2			0.3			0.3		17.0	1.5		18.8	3.6		21.1	5.9		33
34		21.6	5.7		22.1	6.4		22.8	6.9		22.8	6.6		21.9	5.3		19.8	2.9		17.0	24.0			21.1	34
35		20.6	3.9		18.5	1.7			23.2	6.5		21.3	4.8		20.3	4.2		20.3	4.5		21.2	5.6		22.4	35
36	17.5	1.8		18.7	3.3		20.6	5.2		22.4	6.8		23.4	7.5		23.3	7.1		22.3	5.8		20.6	4.0		36
37											20.8	3.6		17.7	0.9			23.4	7.1		23.0	7.0		23.2	37
38																				22.6	6.2		21.3	4.8	38

B.4. Eclipse magnitudes in "fingers" with 12^{f} = lunar diameter

B4. Lunar Eclipses Visible in Babylon: Magnitudes

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	Mag	EP
1	11.2		9.2	8.0			3.6	1.9																	1
2	-12.4		-10.2	-9.4		-8.0	-7.6		-7.0	-6.8	-6.7	-6.7		-6.5	-6.4		-5.8	-5.3		-3.7	-2.8		-0.4		2
3	-16.9		-18.5	-19.4		-22.0	20.9			16.1	14.4		10.9	9.2			4.7			1.2	0.2				3
4	16.3	17.6		19.7	20.6		21.8	-22.1		-21.6	-21.4		-21.2	-21.2		-21.0	-20.6		-19.8	-19.1		-17.2	-16.0		4
5		-2.5	-3.1		-4.9	-6.0		-8.6	-10.1			-15.0			-20.2	-21.7		20.6	19.3		16.9	16.1		14.6	5
6				2.4			5.2	5.8		6.5	6.7		7.0	7.1		7.3	7.6		8.3	8.9		10.6	11.6		6
7													-0.8			-5.6			-10.1	-11.4		-13.6	-14.5		7
8																									8
9	11.6		11.5	11.4	11.2	10.9	10.4	9.8	9.0		7.0	5.8			1.2										9
10	-18.0	-16.2			-11.3	-9.7		-7.1	-6.0		-4.2	-3.6		-2.8	-2.5		-2.4	-2.5		-2.5		-2.5	-2.4		10
11	-17.8	-17.8		-17.8	-17.8		-18.4	-18.8		-20.3	-21.2			18.8	17.3		13.7	11.8			5.9	4.0			11
12	9.7		13.2	14.9			19.7			-21.8			-19.3		-18.4	-18.1		-17.9	-17.9		-17.8	-17.8	-17.6	-17.4	12
13	-1.6	-1.6		-1.7	-1.8		-2.2	-2.5		-3.8	-4.8		-7.1	-8.4	-10.1		-13.6	-15.4			-21.2	20.8		17.2	13
14					0.7			5.2			8.8			11.2		12.0	12.2		12.4	12.4		12.4	12.4	12.5	14
15																				-2.9	-4.8			-10.2	15
16	-10.9		-8.2	-6.6			-1.4																		16
17	8.2		7.3			7.0		7.1	7.1		7.1	7.0		6.4	5.9		4.4	3.5		1.1					17
18		18.4	19.7		-21.2	-19.6			-13.8	-11.9			-6.5	-4.8		-2.0	-0.8								18
19	-20.4	-21.1		-22.0	-22.3		-22.6	-22.6	-22.6	-22.6		22.4	22.2		21.5	20.9		19.3	18.2		15.8	14.4		11.3	19
20		1.3	2.6		5.8	7.6			13.3	15.2		19.1	20.9	-21.4		-18.2	-16.9		-14.9	-14.0		-13.1	-12.7		20
21		-5.9	-6.6		-7.4	-7.7		-7.8		-7.8	-7.7		-7.8	-7.9		-8.6	-9.1		-10.8	-11.8		-14.3	-15.7		21
22												2.8	4.6		7.9	9.4			13.0			15.2		16.0	22
23																									23
24		-11.2	-10.9		-9.7	-9.0		-7.0	-5.8			-1.6											**1.2**	**1.6**	24
25	13.9	12.2		9.0	7.7		5.5	4.6			3.0			2.4		2.2	2.0			1.1	0.6				25
26	18.1	18.4	18.6	19.1		20.4	21.2		-22.1	-20.8		-17.9	-16.4			-11.4			-6.6	-5.0		-2.4	-1.2		26
27	-13.3			-18.5	-20.0		21.4	20.3			18.4	18.0	17.8	17.6	17.6	17.6	17.5	17.5	17.4	17.2		16.3	15.7		27
28		3.5	3.6	3.8	4.2		5.3	6.1		8.2	9.4			13.8			18.7	20.4		-21.1	-19.6			-15.5	28
29				-2.4			-6.7	-7.8		-9.5	-10.1		-10.8	-11.0		-11.2	-11.3	-11.3	-11.4		-11.9	-12.2		-13.4	29
30																	0.5		4.1	5.9			10.7	12.1	30
31	6.4		3.1	1.6																					31
32		-8.0	-7.8		-7.7	-7.4		-7.0	-6.6		-5.3	-4.4		-2.3	-1.0										32
33	-22.6	21.4		18.0	16.3		13.1	11.5			7.3			4.3		3.1	2.6		2.2	2.0		1.9	1.8		33
34		20.5	20.8		20.9	21.0		21.4	21.6		-21.5	-20.9		-19.2	-18.1		-15.6	-14.2		-11.0	-9.4			-4.4	34
35		-7.3	-8.9		-12.1	-13.8			-18.6	-20.0		21.8	20.6		18.7	18.0		16.9	16.7		16.3	16.2		16.1	35
36	5.3	5.6		6.1	6.2		6.4	6.5		7.0	7.3		8.4	9.2		11.3	12.5		15.2	16.8		20.0	21.7		36
37											-3.7	-5.3		-7.8	-8.9			-11.2	-11.5		-12.1	-12.2		-12.5	37
38																				1.1	2.5		5.6	7.2	38

*B.5. Sidereal longitude, λ*m, of the moon at syzygy, calculated as*

$$\lambda^{*}m = \lambda_{\text{trop}} + 10^{\circ} - 1.3825 \times (500 + JY)/100,$$

where λ_{trop} *is the tropical longitude and JY = the Julian year of the event*

B5. Lunar Eclipses Visible in Babylon: Sidereal longitudes, λ*m, of the Moon at Syzygy

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	λ*m	EP
1	144		165	175			206	216																	1
2	314		335	346		367	18		40	51	62	72		94	105		127	138		159	170		190		2
3	132		154	164		185	195			226	236		257	267			297			329	339				3
4	304	314		335	346		367	18		40	50		72	83		105	116		138	148		169	180		4
5		132	143		164	174		195	206			236			267	277		297	308		329	339		360	5
6				324			356	367		28	39		61	72		94	105		126	137		158	169		6
7													236			267			298	308		329	339		7
8																									8
9	70		92	103	113	124	135	146	156		177	188			219										9
10	244	254			285	295		315	326		347	358		19	30		52	63		85		107	117		10
11	59	70		91	102		124	135		156	167			198	208		229	239			269	279			11
12	234		254	264			295			326			358		19	30		52	63		85	96	107	118	12
13	47	58		80	91		113	124		145	156		177	187	197		218	228			258	269		289	13
14					264			295			326			358		19	30		52	63		85	96	107	14
15																				238	248			278	15
16	185		206	216			247																		16
17	355		16			49		71	82		104	115		136	147		168	179		199					17
18		185	195		216	226			256	267			297	307		328	339								18
19	344	355		16	27		49	60	71	82		104	115		136	147		168	179		199	210		230	19
20		174	184		205	215			246	256		276	286	297		317	328		349	359		21	32		20
21		344	355		16	27		49		71	82		104	115		136	147		168	179		200	210		21
22												266	276		296	307			338			10		31	22
23																									23
24		123	133		155	165		186	197			227											10	21	24
25	284	295		315	326		347	357			29			62		84	95			127	138				25
26	101	112	123	133		155	165		186	197		217	227			258			289	299		320	330		26
27	274			304	315		335	346			18	29	40	51	62	73	84	94	105	116		138	148		27
28		101	112	123	134		155	166		186	197			228			258	268		289	299			330	28
29				294			325	335		356	367		29	39		61	72	83	94		116	127		148	29
30																	248		268	278			309	319	30
31	225		245	255																					31
32		49	59		81	92		114	125		146	157		178	188										32
33	215	225		245	255		276	286			317			348		10	21		42	53		75	86		33
34		37	48		70	81		103	113		135	146		167	177		198	208		229	239			269	34
35		215	225		245	255			286	296		317	327		348	359		21	31		53	64		86	35
36	15	26		48	59		80	91		113	124		146	156		177	188		208	219		239	249		36
37											296	306		327	338			9	20		42	53		75	37
38																				208	219		239	249	38

C. Intervals Between Eclipses Visible in Babylon (time-degrees (^{uš}) in Excess of Whole Days)

C.1. 223 months

C1. Lunar Eclipses Visible in Babylon: Lengths of preceding **223m** xs 6585 days in time-degrees

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	223m	EP
1				106				99																	1
2				123			128			132	132	132			131			125			118				2
3				112			107				104			105							118				3
4		113			118			123			127			127			123			116			108		4
5			123			117			113							112			116			122			5
6								115			120			122			120			115			108		6
7																				116			121		7
8																									8
9				132	130	129	127	124	122			115													9
10		102				107			114			121			128			132					130		10
11		128			126			121			113				103			**98**				**98**			11
12				110												131			**134**			**134**	133	132	12
13		122			122			118			111			104	101			**97**				**97**			13
14																	130			133			132	131	14
15																					102				15
16				110																					16
17									131			129			125			119							17
18			107			101				98				101			107								18
19		125			131			**134**	**134**	**135**			132			127			121			115			19
20			104			98				**96**			98	100			106			114			121		20
21			125			130					134			131			125			118			111		21
22													103			108									22
23																									23
24			126			121			116															129	24
25		101			105			112									121				115				25
26		132	131	129			123			117			113							114			118		26
27					103			110				120	122	123	124	125	125	124	124	123			118		27
28			132	130	128			122			115							107			108				28
29								114			122			128			131	131	131			127			29
30																				99				104	30
31				111																					31
32			122			121			117			110			104										32
33		114			112			113									128			131			131		33
34			122			124			122			117			111			107			105				34
35			111			108				109			113			119			123			126			35
36		124			129			130			127			122			116			112			111		36
37												103			108				117			121			37
38																					114			112	38

C.2. 235 months

C2. Lunar Eclipses Visible in Babylon: Lengths of preceding **235m** xs 6939 days in time-degrees

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	235m	EP
1																									1
2																									2
3				130																					3
4		333		323	318		308	303		292	287		276			259			241			220			4
5		220			240			261				289			308			325			339	343			5
6								191			173			157			142			130			119		6
7													5			**10**			**12**	**12**		**11**	**11**		7
8																									8
9																									9
10																									10
11		298		288	282		271	265		252															11
12			280				306			323			336		343	346		352	354		359		3	5	12
13		184			168			152			137				120			112				108			13
14					4			**10**			**12**			**11**		**10**	9		7	5		2	0	358	14
15																									15
16																									16
17																									17
18		256			234																				18
19		320		328			339		345	349		354	356		1	3		7	9		11				19
20			139			126				113				**108**			110								20
21		10	**11**		**11**	**11**		10		9	8		6	4		1	359		353	350		343	339		21
22													159		172									232	22
23																									23
24																							330		24
25																									25
26			349	352		357	0		5	7			**11**												26
27					**109**			**109**				115			122		129	132				150			27
28		10	9	8	7		5	3		358	355			344			330			314	308			289	28
29								176					207	214		226	232	238	244		256			274	29
30																			212						30
31																									31
32																									32
33		7		**10**	**11**																				33
34			111			117			125			137			152										34
35		359	356		349	345			331			315			299			282			265			247	35
36				195			213			232			251			271			291			310			36
37											214			193					162			147			37
38																				5	7		10	11	38

C.3. 12 months

C3. Lunar Eclipses Visible in Babylon: Lengths of preceding **12m** xs 354 days in time-degrees

SC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	12m	EP
1																									1
2																									2
3	6		18	24			44																		3
4	220			201			180			161	155			139			126			114			100		4
5			112			138															221				5
6				102			76	68		53	46		35	30		22	19		15	13		11	11		6
7																**258**			256			250			7
8																									8
9																									9
10																									10
11	170			156	152		144	141			128				108										11
12	164														215			220				230	233		12
13	62	56		46	41		34	31		26	24			19	18		15	14			11	11			13
14																239			233			228	227	227	14
15																					65			82	15
16																									16
17																									17
18			137																						18
19	195							211	214			224			236			248							19
20		35	33		27	25			17	15			9	7		4	3								20
21		246			241			236		234			234			234			233			227			21
22												56	61			75			88			101			22
23																									23
24																								207	24
25																									25
26		218	222			237			250			**258**											97		26
27	17			7	4		**-1**	**-3**			**-5**			**-2**		4	8			24					27
28		238	239	239			241			240										200					28
29				50			62	66			77		86	91		101	107	114	120			143			29
30																	120			97				64	30
31																									31
32																									32
33	253			**260**																					33
34		**-10**	**-10**		**-7**	**-4**		3	9		20	27		41	48										34
35		246			237											167			146			127			35
36		63			78			97			119			144			170			192					36
37												95			70			46	39		26	21		13	37
38																				253			259		38

C.4. 6 months

C4. Lunar Eclipses Visible in Babylon: Lengths of preceding **6m** xs 176 days in time-degrees

SC>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	<SC
beg>	-746	-728	-710	-692	-674	-656	-638	-620	-602	-584	-566	-548	-530	-512	-494	-476	-458	-440	-422	-404	-386	-368	-350	-332	<beg
EP	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	6m	EP
1																									1
2	351		371	388			461																		2
3	376		367	356		324	303			224	195			111			45			**17**	**17**				3
4	565			564			597			656	679		726	749			801			**817**					4
5		263			278			261								141			121			139			5
6								527								602			614			592			6
7													405			377			362	364		378	391		7
8																									8
9																									9
10	194				80	59		28	20		**19**	25			72										10
11	696	722			792			**832**			**829**			781	756		696	662				524			11
12	188			132			94			91					179			278			389	426			12
13	594			633			660			655			610		558			456			342	305		237	13
14					329			312			326			378			464					643		709	14
15																				208				93	15
16																									16
17	28		49																						17
18			808			732			634					485			432								18
19		91			163				300	336			438			515									19
20		664			584				438	399		325	291			209			167			165			20
21		302	323		377	409				556			663	694		745	763		786	789		782	772		21
22													118			51			23			39			22
23																									23
24																									24
25		551			489			461																	25
26	355	387		441			495									469				444					26
27	381			285			224					211	219			255			291	300		309	309		27
28				674	699		737	748			754			722			671	653		620					28
29				97			45	37		37	44			89			156	181			248			294	29
30																	684		620					490	30
31																									31
32			538																						32
33		449			421			399			401			438											33
34		261			292			324			339			323			274			201					34
35		705	694		666	650			605			580			585			621			679			740	35
36		78			132					238			278			288			266			225			36
37											570			513					493			516			37
38																					469			443	38

D. Sigma Values Arranged in 14-month Cycles from LUSIX2 (P.J. Huber); Data in Bold are Those Required for the Analysis Described in the Text with $q = 9$. Sig-7 = Sigma Values for That Month Averaged over 7 Steps and Associated with Median syzygy

D.1.a. Sigma (–625 to –565)

D.1a. Sigma values (-625 to -565)

GN2 (mo 1)	JY	mo 1	mo 2	mo 3	mo 4	mo 5	mo 6	mo 7	mo 8	mo 9	mo 10	mo 11	mo 12	mo 13	mo 14
4642	-625	28.0	28.9	29.1	25.4	22.8	21.2	21.2	22.8	25.1	26.7	26.6	26.4	27.5	30.3
4656	-624	30.9	30.4	27.0	23.0	21.8	22.3	23.7	25.0	23.8	23.7	24.9	27.2	31.3	31.8
4670	-623	29.9	26.7	24.0	24.4	25.7	25.6	23.1	21.7	21.4	24.3	29.5	32.3	30.5	28.1
4684	-622	26.4	26.2	28.2	29.3	25.2	22.1	19.9	21.0	25.1	29.5	30.1	28.8	26.3	27.2
4698	-621	29.4	32.1	30.0	25.3	21.1	20.2	22.5	26.3	27.6	25.7	24.2	25.4	29.2	35.5
4712	-619	33.3	29.0	23.1	21.6	22.5	25.3	26.0	24.4	21.9	22.2	27.0	31.5	36.4	31.3
4726	-618	27.2	24.0	24.9	26.5	27.1	25.0	21.1	20.0	22.3	28.4	31.9	32.8	28.0	26.1
4740	-617	26.7	29.6	30.3	28.3	22.7	20.0	20.2	23.9	28.8	28.9	26.5	25.9	27.0	29.5
4754	-616	33.2	31.2	25.4	22.6	21.1	22.4	25.4	26.2	24.9	23.6	25.3	28.5	32.2	32.1
4768	-615	30.1	25.3	23.4	23.3	25.0	25.4	23.7	21.9	22.2	25.8	29.5	32.4	30.9	28.7
4782	-614	26.2	26.3	26.7	26.7	24.9	21.7	20.7	22.2	24.7	28.9	29.0	28.7	27.5	27.8
4796	-613	28.5	29.5	26.6	24.4	21.6	21.0	22.1	24.8	25.7	26.4	26.5	27.4	29.0	30.6
4810	-612	31.0	27.4	24.5	22.6	22.0	23.0	23.5	24.2	24.1	24.7	27.0	29.3	30.7	31.4
4824	-610	28.1	25.6	24.2	23.4	23.4	23.4	22.7	22.5	23.4	25.6	29.0	30.2	31.0	28.6
4838	-609	27.0	26.0	25.4	24.8	23.3	22.3	21.5	22.2	24.5	27.8	28.9	29.9	28.4	28.0
4852	-608	27.7	27.4	26.2	24.7	22.2	21.2	21.6	23.9	26.8	27.1	27.9	27.8	28.6	29.3
4866	-607	29.4	27.7	25.5	22.9	21.4	22.1	24.1	25.6	24.9	25.4	26.2	28.9	30.6	32.3
4880	-606	30.1	26.0	23.4	22.6	23.8	24.6	23.8	23.1	23.4	24.9	29.0	31.0	32.2	29.6
4894	-605	26.9	24.7	25.2	26.1	24.4	22.5	21.0	21.9	24.4	29.3	30.3	30.1	28.0	26.7
4908	-604	26.9	29.0	27.7	24.9	21.0	20.2	21.3	25.0	28.9	29.1	26.7	26.3	27.6	31.9
4922	-602	32.4	29.5	23.7	21.0	20.6	23.0	26.1	26.3	25.0	23.9	26.0	30.0	34.7	33.1
4936	-601	28.0	23.3	22.5	24.3	25.7	25.5	23.7	21.3	22.7	27.0	34.2	34.0	30.8	25.6
4950	-600	24.5	25.9	28.2	27.3	24.6	20.6	20.3	23.6	28.6	33.3	30.0	27.5	26.1	28.5
4964	-599	30.7	31.0	26.1	22.5	19.7	21.2	26.0	28.1	28.3	25.7	25.3	27.7	32.4	33.9
4978	-598	31.3	25.5	21.5	21.5	24.1	25.7	25.2	24.0	23.0	25.1	28.6	34.1	33.4	29.9
4992	-597	25.2	23.6	25.1	25.9	25.0	22.3	21.0	22.8	25.9	29.6	31.0	30.9	27.3	26.3
5006	-596	26.9	27.7	27.0	23.4	20.9	20.7	22.1	26.4	28.9	28.5	27.2	27.3	27.9	30.3
5020	-595	30.4	26.9	23.2	21.0	21.2	23.3	24.6	26.5	25.7	25.8	27.3	29.3	32.1	31.4
5034	-593	27.5	23.8	22.8	22.8	23.2	23.6	23.6	23.4	24.4	27.1	29.6	32.3	29.9	28.0
5048	-592	25.4	24.7	24.2	23.9	23.3	22.2	22.1	23.2	26.5	28.7	31.1	29.5	28.4	27.2
5062	-591	26.6	26.0	25.1	23.3	21.6	21.4	22.6	24.5	27.2	29.2	28.5	28.3	28.3	28.4
5076	-590	27.8	26.4	23.6	21.9	21.3	21.8	23.9	25.7	26.8	27.2	27.9	28.9	29.8	29.5
5090	-589	27.7	25.1	22.6	21.8	22.4	23.5	24.8	24.5	25.0	26.4	29.1	30.6	30.4	28.4
5104	-588	25.7	23.8	23.3	23.6	24.3	23.1	22.8	23.1	25.4	28.9	32.3	31.5	28.2	26.2
5118	-587	25.5	25.6	26.2	23.9	22.1	20.8	22.3	24.6	28.4	30.5	29.3	27.8	27.2	28.9
5132	-586	29.8	27.5	24.3	20.9	20.4	22.8	26.5	28.1	27.1	26.2	26.3	28.5	32.3	33.2
5146	-584	27.3	23.6	21.0	21.8	24.7	25.0	25.3	24.1	23.6	26.1	32.4	34.6	32.6	26.3
5160	-583	23.7	23.1	25.1	26.2	25.0	21.7	20.7	23.4	27.2	33.3	33.2	29.5	25.7	25.6
5174	-582	26.8	28.8	27.1	22.4	19.9	20.3	24.7	30.8	30.7	29.0	26.1	26.4	29.2	31.6
5188	-581	30.0	25.9	21.4	19.9	22.0	26.4	27.8	26.2	24.9	25.4	29.4	33.2	34.3	29.0
5202	-580	24.5	21.8	22.1	25.0	26.0	24.5	22.3	22.7	25.0	31.6	34.4	33.3	28.0	24.4
5216	-579	24.1	25.9	26.6	24.8	21.5	20.5	22.6	26.0	31.6	32.1	28.7	27.1	26.7	27.9
5230	-578	30.1	27.5	23.0	20.6	20.6	23.8	26.6	27.6	28.3	26.7	26.9	29.5	31.2	29.4
5244	-576	27.2	22.9	21.2	22.1	23.8	24.6	24.9	24.3	25.2	27.1	31.3	32.7	29.8	25.9
5258	-575	24.0	23.2	23.5	23.7	23.6	22.7	22.7	24.5	27.0	31.4	32.2	29.7	26.9	25.8
5272	-574	25.1	25	24.1	22.3	21.4	21.7	24.1	26.5	28.5	29.5	29	28.1	27.9	27.4
5286	-573	26.7	24.7	22.4	21.4	21.6	22.9	25.6	26.8	27.7	28	28.2	28.6	29.3	28.3
5300	-572	25.5	23.6	21.9	21.6	22.9	24.8	25.1	25.7	26.7	27.9	29.2	30.5	29.7	27.5
5314	-571	24.5	22.8	22.7	23.6	24.1	23.5	23.9	25	27.1	29.2	31	30.1	28	25.9
5328	-570	24.3	24.7	24.7	23.1	22.6	22.1	23.1	26.2	28.8	30.3	29.6	28	26.7	26.4
5342	-569	27.3	25.2	23.7	21.1	21.1	22.4	25.9	27.7	29.6	28.7	27.4	27.6	29.1	29.9
5356	-567	27.5	22.9	20.9	20.8	23	26.2	26.1	26.2	25.7	26.9	30.4	31.4	31.1	27.3
5370	-566	23.3	21.6	22.5	24.9	25.3	24.3	22.7	23.7	27	32.5	35.5	29.9	26.3	23.6

D.1.b. Sigma (–460 to –400)

D.1b. Sigma values (-460 to -400)

GN2 (mo 1)	JY	mo 1	mo 2	mo 3	mo 4	mo 5	mo 6	mo 7	mo 8	mo 9	mo 10	mo 11	mo 12	mo 13	mo 14
6686	-460	23.4	23.9	25.6	28.6	30.3	30.3	29.0	26.8	25.2	24.3	24.0	23.6	22.6	22.2
6700	-459	22.5	24.4	27.7	29.6	31.1	28.9	27.4	26.6	25.7	25.8	24.1	22.6	21.2	21.4
6714	-458	23.6	26.8	29.3	29.2	28.3	27.9	28.3	29.8	27.8	25.7	23.1	20.9	21.4	23.7
6728	-456	26.5	26.6	26.3	26.5	28.2	30.0	32.5	28.9	25.4	22.9	21.5	22.7	24.8	24.9
6742	-455	24.2	23.5	24.7	28.7	33.4	33.3	30.5	25.3	23.7	23.8	25.6	25.2	23.4	21.3
6756	-454	21.9	25.3	29.2	32.9	31.4	28.5	25.8	26.6	27.8	28.1	25.3	21.3	19.9	22.0
6770	-453	26.5	29.7	29.6	27.7	26.1	27.2	30.7	33.4	28.1	23.9	20.3	20.6	23.6	27.2
6784	-452	27.5	25.1	24.3	25.7	31.3	34.4	33.6	26.6	23.1	21.5	23.1	25.9	26.1	23.5
6798	-451	21.7	23.3	26.6	33.7	34.6	31.7	25.7	24.0	24.5	27.0	27.1	23.6	20.7	20.4
6812	-450	23.9	30.4	32.5	31.2	27.5	25.9	27.2	29.1	29.5	26.5	22.0	19.7	21.0	25.1
6826	-449	28.3	28.5	27.0	25.9	28.2	31.1	32.6	29.0	25.0	21.6	20.9	23.1	25.1	26.0
6840	-447	24.4	24.0	25.4	30.4	33.4	33.4	28.6	25.0	23.2	23.7	24.6	24.9	23.1	21.9
6854	-446	23.0	25.7	29.0	32.6	29.9	28.1	26.3	26.0	26.3	26.3	23.8	21.4	21.0	23.1
6868	-445	25.8	28.1	29.1	28.4	27.8	28.2	28.8	29.0	27.0	23.5	21.3	21.4	22.6	24.3
6882	-444	26.2	26.8	26.8	27.5	28.9	30.9	29.4	27.6	24.1	22.3	22.2	22.5	23.6	23.4
6896	-443	24.5	25.5	27.1	29.1	31.8	30.1	28.1	25.3	23.7	23.2	23.3	22.6	23.0	22.9
6910	-442	23.8	26.2	28.6	31.5	30.2	28.7	27.0	25.5	25.1	23.8	23.6	22.2	21.8	22.7
6924	-441	25.3	27.6	29.3	29.3	28.6	27.8	26.8	27.1	26.3	23.9	22.3	21.1	21.9	23.6
6938	-439	26.3	28.4	27.6	27.9	28.2	28.8	29.3	27.8	25.4	22.2	21.1	21.7	24.9	25.1
6952	-438	25.8	25.8	26.7	28.5	31.9	30.7	28.5	25.6	22.7	22.2	23.1	24.8	24.1	23.0
6966	-437	23.2	26.0	28.7	33.0	32.7	28.0	25.6	24.5	25.5	26.1	24.7	22.0	21.1	21.9
6980	-436	25.9	31.0	32.1	30.6	26.9	26.2	27.5	29.3	28.0	23.4	20.8	20.0	22.3	26.6
6994	-435	29.4	28.6	27.0	26.2	28.9	31.5	31.5	27.9	23.2	20.6	21.0	24.0	26.9	26.6
7008	-434	24.2	23.9	26.5	31.9	34.6	31.0	26.5	22.4	22.1	23.9	26.2	25.7	22.7	21.3
7022	-433	23.1	29.6	33.6	34.2	28.7	25.6	24.5	26.7	27.9	27.8	23.1	20.1	20.2	24.2
7036	-432	30.6	32.0	30.5	26.6	26.3	28.0	31.2	29.7	26.4	21.1	19.7	21.4	25.8	29.6
7050	-430	27.6	25.5	25.7	28.7	31.8	33.5	29.1	23.7	21.5	21.4	23.8	26.8	25.5	23.5
7064	-429	23.4	27.0	30.8	33.9	31.2	28.4	24.4	23.8	24.5	26.6	24.9	22.5	21.3	22.9
7078	-428	27.8	31.3	32.6	29.6	27.8	26.2	26.5	27.6	26.9	23.6	20.9	20.8	23.2	26.0
7092	-427	27.9	28.4	27.8	27.6	28.7	29.7	29.5	25.7	23.0	21.2	21.7	23.0	25.5	25.8
7106	-426	25.7	26.3	28.3	30.3	31.7	29.8	26.2	23.7	22.5	22.4	23.8	23.6	23.6	23.6
7120	-425	25.0	27.2	30.7	30.9	30.1	27.1	25.6	24.3	24.1	23.7	23.1	22.2	22.3	24.0
7134	-424	26.4	30.0	30.2	30.0	27.9	27.1	26.2	25.8	24.6	22.7	21.8	21.6	22.7	25.4
7148	-423	28.7	28.7	29.1	28.4	28.1	28.2	27.7	25.6	23.8	21.6	21.2	22.4	24.6	27.0
7162	-421	26.8	27.3	27.8	28.5	29.9	29.5	28.1	24.4	22.3	21.7	22.7	24.6	25.0	25.0
7176	-420	25.5	26.9	28.6	31.1	30.7	28.5	25.6	23.2	23.2	24.4	24.3	23.6	22.6	23.5
7190	-419	26.1	30.6	31.3	30.6	28.0	25.9	25.1	26.3	25.9	24.3	21.6	21.0	21.9	25.9
7204	-418	30.4	31.2	28.4	27.1	27.2	28.1	29.7	27.9	23.2	20.8	20.5	22.6	25.9	28.5
7218	-417	27.5	25.9	26.4	29.4	33.2	31.6	27.0	22.3	20.8	22.4	24.1	26.4	24.7	23.6
7232	-416	24.2	27.9	32.1	34.2	30.8	26.5	23.0	23.4	25.9	26.4	23.9	22.0	21.0	23.1
7246	-415	29.6	35.5	31.6	28.3	25.4	25.4	27.7	28.6	25.7	22.6	19.5	21.4	26.2	30.2
7260	-413	31.2	27.8	26.2	26.6	30.7	31.9	30.2	24.5	20.4	20.1	22.9	26.1	27.7	26.6
7274	-412	24.7	25.9	28.9	31.8	33.1	29.0	23.7	21.5	22.6	24.5	25.8	23.8	22.2	23.2
7288	-411	27.0	31.1	34.1	31.2	27.6	24.6	24.4	26.6	26.3	23.7	21.1	21.0	22.7	28.0
7302	-410	31.3	30.5	29.2	26.6	26.6	28.4	28.6	26.0	22.2	20.4	21.4	23.5	26.0	29.0
7316	-409	27.6	26.7	27.5	29.1	31.3	30.1	25.8	22.3	21.2	22.0	24.5	24.9	25.2	24.9
7330	-408	25.9	28.2	30.7	31.0	30.3	26.2	23.5	22.9	23.0	23.7	22.7	23.0	23.3	24.6
7344	-407	28.4	30.9	30.9	29.0	27.1	25.5	25.3	24.5	23.1	22.8	21.7	22.4	23.8	27.7
7358	-406	29.2	30.5	28.8	28.0	27.3	27.0	26.1	24.8	22.6	21.3	21.5	23.5	25.3	27.4
7372	-404	28.6	28.1	28.4	28.8	28.9	27.6	25.7	22.9	21.8	21.6	22.5	24.3	25.5	26.4
7386	-403	26.8	27.6	29.8	30.3	28.8	26.6	24.1	22.5	22.2	22.9	23.7	24.3	24.2	25.1
7400	-402	27.1	30.1	31.0	31.0	27.3	25.0	23.9	23.7	23.8	23.8	22.2	22.2	23.8	26.0
7414	-401	29.7	30.6	30.1	28.0	26.2	25.9	26.1	25.8	22.7	21.2	20.9	23.4	25.9	28.6

D.2.a. Sig-7 (–622 to –565)

D.2a. Sig-7 values (-622 to -565)

GN2 (mo 1)	JY	mo 1	mo 2	mo 3	mo 4	mo 5	mo 6	mo 7	mo 8	mo 9	mo 10	mo 11	mo 12	mo 13	mo 14
4684	-622	29.3	28.2	26.6	25.1	23.7	23.1	22.5	23.0	23.9	25.8	27.7	29.2	29.9	30.0
4698	-621	29.1	28.3	26.8	25.5	23.7	22.9	22.4	23.2	24.4	26.1	27.7	29.1	29.8	29.9
4712	-619	29.4	28.4	26.6	25.4	23.6	22.9	22.6	23.4	24.6	26.1	27.8	29.3	29.9	30.0
4726	-618	29.5	28.2	26.5	25.3	23.5	22.9	22.7	23.4	24.7	26.3	27.8	29.3	30.0	30.1
4740	-617	29.4	28.2	26.3	24.9	23.5	22.9	22.8	23.6	24.6	26.2	27.6	29.3	30.2	30.1
4754	-616	29.3	27.8	25.8	24.8	23.6	23.0	22.7	23.3	24.4	26.3	28.0	29.6	30.1	29.4
4768	-615	29.0	27.6	26.0	24.9	23.5	22.6	22.4	23.3	24.7	26.7	28.0	29.3	29.3	29.5
4782	-614	29.1	27.8	25.9	24.5	23.0	22.4	22.6	23.7	24.8	26.3	27.5	28.9	29.8	29.8
4796	-613	**29.2**	**27.3**	25.2	24.0	23.0	22.7	22.8	23.4	**24.2**	**26.1**	27.9	29.5	30.0	29.6
4810	-612	**28.4**	**26.8**	25.3	24.3	23.2	22.6	22.3	23.1	**24.5**	**26.6**	28.3	29.4	29.4	29.2
4824	-610	**28.3**	**27.1**	25.6	24.2	22.7	22.1	22.3	23.6	**24.9**	**26.6**	27.8	28.9	29.4	29.7
4838	-609	**28.8**	**27.1**	25.1	23.6	22.5	22.5	22.8	23.8	**24.7**	**26.0**	27.8	29.2	30.1	30.0
4852	-608	**28.6**	**26.4**	24.9	23.9	22.9	22.7	22.6	23.3	**24.5**	**26.4**	28.3	29.6	29.9	29.4
4866	-607	**28.0**	**26.6**	25.4	24.2	22.8	22.3	22.3	23.5	**25.2**	**27.0**	28.3	29.2	29.5	29.5
4880	-606	**28.6**	**27.2**	25.3	23.9	22.4	22.3	22.8	24.0	**25.4**	**26.8**	27.9	29.1	30.0	30.1
4894	-605	**28.8**	**26.8**	24.9	23.8	22.7	22.7	23.1	23.9	**25.2**	**26.7**	28.6	29.7	30.4	29.8
4908	-604	**28.3**	**26.6**	25.2	24.2	23.1	22.6	22.9	23.8	**25.4**	**27.6**	28.9	29.7	30.0	29.7
4922	-602	**28.5**	**27.1**	25.3	24.1	22.8	22.5	23.2	24.2	**25.9**	**27.6**	28.8	29.5	30.3	29.9
4936	-601	**28.7**	**27.0**	25.0	23.9	22.9	22.7	23.4	24.3	**25.8**	**27.6**	28.7	30.0	30.4	29.9
4950	-600	**28.4**	**26.8**	25.0	23.9	23.0	22.6	23.4	24.4	**26.1**	**27.7**	28.8	30.1	30.3	29.9
4964	-599	**28.4**	**26.6**	24.9	23.7	22.9	22.7	23.5	24.6	**26.1**	**27.6**	28.9	30.2	30.4	29.7
4978	-598	**28.1**	**26.3**	24.8	23.7	23.0	22.8	23.3	24.7	**26.2**	**27.9**	29.1	30.1	30.0	29.4
4992	-597	**28.1**	**26.3**	24.8	23.5	22.7	22.5	23.3	25.0	**26.4**	**27.9**	28.4	29.9	29.9	29.8
5006	-596	**28.2**	**26.2**	24.3	23.0	22.5	22.7	23.5	24.9	**26.1**	**27.2**	28.6	30.2	30.2	29.6
5020	-595	**27.6**	**25.5**	24.1	23.1	22.8	22.7	23.0	24.4	**25.9**	**27.7**	29.0	30.2	29.6	28.8
5034	-593	**27.1**	**25.6**	24.4	23.2	22.4	22.2	22.8	24.6	**26.5**	**28.0**	28.9	29.5	29.1	28.7
5048	-592	**27.5**	**25.8**	24.1	22.6	22.0	22.4	23.4	24.9	**26.4**	**27.6**	28.7	29.5	29.5	29.0
5062	-591	**27.3**	**25.2**	23.5	22.6	22.5	22.7	23.5	24.4	**25.9**	**27.6**	29.4	30.1	29.6	28.4
5076	-590	**26.6**	**25.1**	24.0	23.0	22.6	22.3	23.2	24.1	**26.2**	**28.3**	29.7	29.8	28.9	28.1
5090	-589	**26.9**	**25.6**	24.2	22.8	22.2	22.2	23.6	24.8	**26.6**	**28.2**	29.2	29.3	29.2	28.8
5104	-588	**27.2**	**25.4**	23.7	22.5	22.4	22.6	24.0	24.9	**26.2**	**27.8**	29.4	30.0	29.8	28.7
5118	-587	**26.8**	**25.0**	23.7	22.9	22.9	22.7	23.8	24.8	**26.2**	**28.4**	30.1	30.2	29.5	28.3
5132	-586	**26.6**	**25.4**	24.2	22.9	22.7	22.5	23.9	25.5	**26.8**	**28.6**	29.8	29.8	29.4	28.6
5146	-584	**27.0**	**25.5**	24.1	22.7	22.6	22.9	24.3	25.8	**26.8**	**28.5**	29.9	30.2	29.9	28.7
5160	-583	**26.8**	**25.2**	23.9	22.9	22.9	23.1	24.2	25.7	**26.7**	**28.9**	30.2	30.5	29.9	28.4
5174	-582	**26.6**	**25.2**	23.9	23.0	22.8	23.0	24.3	25.9	**27.2**	**29.1**	30.1	30.4	29.8	28.3
5188	-581	**26.6**	**25.2**	23.8	23.0	22.8	23.2	24.3	25.8	**27.3**	**29.2**	30.2	30.5	29.7	27.7
5202	-580	**26.6**	**25.1**	23.8	23.0	22.7	23.1	24.2	25.9	**27.6**	**29.3**	30.0	30.2	29.3	27.7
5216	-579	**26.7**	**25.1**	23.6	22.6	22.5	23.3	24.5	26.0	**27.5**	**29.0**	29.9	30.3	29.4	27.7
5230	-578	**26.4**	**24.6**	23.1	22.6	22.7	23.5	24.4	25.4	**27.2**	**29.1**	30.3	30.5	29.3	27.1
5244	-576	**26.0**	**24.4**	23.3	22.8	22.6	23.0	24.1	25.5	**27.6**	**29.5**	30.1	29.9	28.5	27.0
5258	-575	26.1	24.7	23.2	22.4	22.2	23.0	24.5	25.9	27.9	29.0	29.4	29.5	28.8	27.5
5272	-574	26.2	24.2	22.7	22.2	22.6	23.4	24.7	25.8	27.2	28.5	29.7	29.9	29.0	27.2
5286	-573	25.3	23.8	22.9	22.5	22.9	23.2	24.2	25.6	27.3	29.1	30.1	29.7	28.3	26.7
5300	-572	25.3	24.2	23.3	22.4	22.5	22.9	24.3	26.1	27.9	29.3	29.5	28.9	28.2	27.3
5314	-571	25.8	24.1	22.9	22.0	22.4	23.4	24.8	26.3	27.7	28.6	29.3	29.2	28.8	27.5
5328	-570	25.6	23.6	22.7	22.4	22.9	23.7	24.6	25.9	27.5	29.1	30.2	29.4	28.6	27.0
5342	-569	25.2	23.8	23.2	22.8	22.9	23.4	24.3	26.3	28.2	29.7	30.1	29.1	28.2	27.1
5356	-567	25.7	24.3	23.2	22.5	22.5	23.5	25.0	26.9	28.4	29.4	29.8	29.0	28.6	27.4
5370	-566	25.9	24.0	22.9	22.3	22.9	24.1	25.2	26.8	28.2	29.5	30.2	29.6	28.7	27.1

D.2.b. Sig-7 (–460 to –402)

D.2b. Sig-7 values (-460 to -402)

GN2 (mo 1)	JY	mo 1	mo 2	mo 3	mo 4	mo 5	mo 6	mo 7	mo 8	mo 9	mo 10	mo 11	mo 12	mo 13	mo 14
6686	-460	23.9	25.5	27.2	28.2	29.2	29.2	29.3	28.3	26.5	24.6	23.0	22.2	22.4	23.1
6700	-459	24.1	25.2	26.7	28.1	29.7	29.9	29.7	28.0	25.9	24.3	23.3	22.7	22.7	22.9
6714	-458	23.8	25.1	26.9	28.9	30.2	29.9	29.3	27.5	25.9	24.9	23.8	22.7	22.4	22.7
6728	-456	24.1	25.7	27.5	29.0	29.8	29.4	29.2	28.2	26.2	24.9	23.4	22.4	22.4	23.2
6742	-455	24.7	25.9	27.3	28.6	30.0	30.0	29.8	28.2	25.9	24.5	23.3	22.7	22.9	23.4
6756	-454	24.6	25.8	27.1	29.2	30.5	30.4	29.6	27.8	25.8	24.7	23.7	22.9	22.8	23.3
6770	-453	24.6	26.3	27.6	29.5	30.4	30.1	29.4	27.7	26.0	24.8	23.6	22.7	22.8	23.5
6784	-452	24.9	26.5	27.7	29.4	30.4	30.3	29.4	27.7	26.0	24.6	23.5	22.8	22.8	23.6
6798	-451	24.9	26.6	27.8	29.6	30.4	30.3	29.2	27.7	25.9	24.6	23.3	22.7	22.8	23.7
6812	-450	25.0	26.7	27.8	29.6	30.1	30.3	29.2	27.6	25.7	24.4	23.1	22.7	22.9	23.9
6826	-449	24.9	26.4	27.7	29.7	30.4	30.4	29.0	27.0	25.5	24.3	23.3	22.9	22.8	23.5
6840	-447	24.8	26.7	28.1	30.0	30.0	29.9	28.4	27.1	25.7	24.4	23.1	22.4	22.4	23.5
6854	-446	25.2	27.0	28.1	29.3	29.6	29.7	28.7	27.3	25.5	23.9	22.6	22.2	22.8	23.8
6868	-445	25.1	26.4	27.6	29.3	30.0	30.1	28.7	26.8	24.9	23.5	22.8	22.6	22.9	23.5
6882	-444	24.7	26.3	27.9	29.8	30.1	29.6	27.9	26.5	25.1	23.8	23.0	22.3	22.4	23.1
6896	-443	25.0	26.9	28.2	29.5	29.3	28.9	28.0	26.9	25.4	23.6	22.5	21.8	22.7	23.6
6910	-442	25.4	26.9	27.9	28.9	29.6	29.3	28.3	26.8	24.9	23.0	22.4	22.3	23.1	23.6
6924	-441	25.0	26.6	27.8	29.5	30.3	29.3	27.8	26.2	24.7	23.4	22.9	22.4	22.9	23.2
6938	-439	25.0	27.2	28.6	30.0	30.0	28.6	27.5	26.4	25.2	23.5	22.7	22.1	22.7	23.7
6952	-438	25.7	27.7	28.6	29.6	29.6	28.8	28.0	26.8	25.2	23.2	22.4	22.3	23.3	24.2
6966	-437	25.7	27.3	28.3	29.6	30.3	29.1	28.0	26.4	24.7	23.2	22.7	22.8	23.4	24.0
6980	-436	25.4	27.6	28.9	30.3	30.3	28.8	27.6	26.3	25.0	23.7	22.9	22.6	23.2	24.1
6994	-435	26.0	28.1	29.3	30.1	30.0	28.7	27.9	26.6	25.1	23.6	22.7	22.6	23.3	24.7
7008	-434	26.3	28.1	29.2	30.2	30.0	29.1	28.0	26.3	24.9	23.5	22.8	22.9	23.5	24.8
7022	-433	26.3	28.2	29.5	30.3	29.8	29.2	27.8	26.2	24.8	23.5	22.8	22.9	23.5	25.0
7036	-432	26.6	28.3	29.5	30.2	29.9	29.2	27.7	26.0	24.6	23.6	22.8	23.0	23.7	24.9
7050	-430	26.4	28.2	29.6	30.4	29.9	28.9	27.4	25.7	24.6	23.7	22.9	22.9	23.5	24.8
7064	-429	26.6	28.6	29.9	30.1	29.5	28.7	27.3	25.8	24.7	23.4	22.6	22.6	23.6	25.1
7078	-428	26.9	28.2	29.5	29.7	29.7	29.0	27.5	25.5	24.1	22.9	22.6	22.9	23.9	25.1
7092	-427	26.3	28.0	29.4	30.1	29.9	28.8	26.8	24.9	23.9	23.1	22.9	22.9	23.4	24.5
7106	-426	26.4	28.4	29.9	30.1	29.4	28.1	26.6	25.2	24.2	23.1	22.5	22.3	23.3	25.0
7120	-425	26.9	28.5	29.5	29.3	29.2	28.2	27.1	25.3	23.9	22.4	22.2	22.6	23.8	25.3
7134	-424	26.6	27.8	28.9	29.5	29.6	28.6	27.0	24.7	23.4	22.5	22.7	23.0	23.8	24.9
7148	-423	26.3	28.1	29.4	30.0	29.5	28.0	26.4	24.8	23.8	23.0	22.6	22.7	23.2	24.9
7162	-421	27.0	28.8	29.4	29.5	28.8	27.8	26.9	25.4	23.9	22.7	22.2	22.6	23.6	25.6
7176	-420	27.3	28.7	28.8	29.3	29.3	28.4	27.1	25.1	23.4	22.6	22.3	23.2	23.9	**25.6**
7190	-419	**27.0**	28.4	29.1	29.9	29.7	**28.3**	**26.6**	24.7	23.6	23.1	22.6	23.2	23.7	**25.2**
7204	-418	**27.2**	29.3	29.5	29.9	29.3	**27.9**	**26.6**	25.2	23.9	23.2	22.4	23.1	23.9	**25.7**
7218	-417	**27.8**	29.4	29.2	29.6	29.4	**28.3**	**26.9**	25.2	23.6	23.0	22.4	23.3	24.3	**25.9**
7232	-416	**27.7**	29.3	29.3	29.7	29.8	**28.3**	**26.6**	24.9	23.5	23.0	22.6	23.3	24.2	**25.9**
7246	-415	**27.8**	29.3	29.7	29.8	29.7	**28.2**	**26.5**	25.0	23.6	22.9	22.5	23.3	24.3	**26.2**
7260	-413	**27.9**	29.2	29.8	29.7	29.6	**28.2**	**26.4**	24.7	23.4	22.9	22.7	23.5	24.4	**26.2**
7274	-412	**27.9**	29.3	29.9	29.7	29.4	**28.0**	**26.2**	24.7	23.5	22.8	22.7	23.2	24.4	**26.4**
7288	-411	**28.2**	29.4	29.7	29.2	29.3	**27.9**	**26.3**	24.6	23.1	22.4	22.6	23.4	24.8	**26.6**
7302	-410	**28.0**	28.7	29.6	29.3	29.5	**28.0**	**25.9**	24.0	22.7	22.5	22.9	23.5	24.4	**26.3**
7316	-409	**27.7**	29.1	30.0	29.5	29.0	**27.3**	**25.3**	24.1	23.0	22.6	22.7	23.2	24.1	**26.4**
7330	-408	**28.3**	29.4	29.9	29.1	28.4	**27.1**	**25.6**	24.3	22.9	22.2	22.2	23.2	24.5	**26.9**
7344	-407	**28.3**	28.9	29.3	29.0	28.6	**27.3**	**25.6**	23.7	22.3	22.1	22.6	23.7	24.8	**26.4**
7358	-406	**27.7**	28.9	29.6	29.6	28.7	**26.9**	**24.9**	23.4	22.5	22.6	22.7	23.5	24.4	**26.0**
7372	-404	**28.0**	29.4	30.0	29.4	28.0	**26.3**	**25.0**	23.9	22.7	22.5	22.2	23.3	24.5	**26.5**
7386	-403	**28.4**	29.4	29.4	29.0	28.0	**26.8**	**25.3**	23.8	22.7	22.3	22.1	23.4	24.8	26.9

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